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### Introduction

In the last decade a large number of input-output (I/O) models have been built for a variety of regions: cities, states, and multi-state regions. Bourque and Cox [1] catalogue virtually all existing regional I/O models up to 1970, and Giarratani, et al [4] offer a newer and more comprehensive bibliography. Richardson [17] discusses and evaluates much of the empirical and technical literature on I/O models. Despite a number of analytical deficiencies and the high costs of data gathering and processing, I/O models have been useful for practical general equilibrium analysis, specifically for measuring and analyzing interindustry flows as well as determining the impact of changes on the structure of a regional economy.

The efforts and funds allocated to these models considerably exceed those committed to building regional econometric models<sup>1</sup> of which only a handful exists. Efforts are being made to redress this balance because Keynesian type regional econometric models offer the potential of coherently depicting and forecasting the economy's final demand segment, an important component of an I/O model. At the present time, the final demand sector of I/O models is "driven" by a number of ad hoc procedures. It is our contention that the description of a regional aggregate demand sector incorporated within a well-specified state econometric model should be able to predict the I/O final demand vector better than these ad hoc procedures. Taking a broader view, conjoining an I/O model with an econometric model should result in a more fully integrated approach<sup>2</sup> to the economic analysis of a region than has hitherto been the case. It combines a theory of production with a theory of aggregate demand.

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<sup>1</sup>For a discussion of recent trends in regional econometric model building see Glickman [5].

<sup>2</sup>Very little empirical work has been done linking national input-output models to national macroeconomic models. Examples are R. S. Preston [16] for the U. S., and Seguy and Ramirez [21] for Mexico. As far as we know, no work of this kind has been done at the regional level.

In this paper our focus is on the economy of a state. The first objective is to develop a six-sector I/O model of Ohio, using a non-survey technique similar to the "Supply-Demand Pool" (SDP) technique discussed in Schaffer and Chu [19]. The second objective is to discuss how this Ohio I/O model was conjoined with the Ohio econometric model (OEM) described in L'Esperance, *et al.* [7, 8, 10]. An application, discussed in this paper, is to forecast industry employment using this I/O model whose estimates of the final output by industry are provided by the OEM. Our third and final objective is to discuss a 77-sector Ohio I/O model, which is a disaggregation of the six-sector model, and its usefulness in forecasting employment by I/O sector. We also comment on converting these employment estimates into employment by occupation.

The organization of the remainder of this paper is as follows: Section II is a brief review of the structure of an input-output model. Section III contains a brief discussion of the analytics involved: the I/O model itself, the non-survey technique, the conjunction of I/O with an econometric model, and forecasting with the conjoined model. Section III includes a discussion of the 77-sector Ohio I/O model, empirical problems encountered, and various tests of this model as a predictive tool.

#### A Brief Review of the Structure of an Input-Output Model

Before introducing the Ohio I/O model and its derivation from the national model, a brief review of input-output concepts will provide a useful framework for the discussion which follows. I/O techniques focus on the relationships among gross industry outputs, intermediate demands for output (i.e., inputs used in the production process) and final demands for outputs.

Assume there are  $n$  industries producing  $n$  goods purchased by the  $n$  industries and by the consuming sectors for end use.

$$\text{Let } x = \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_i \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \quad \begin{array}{l} = \text{an } n \times 1 \text{ gross output vector} \\ \text{where } x_i = \text{gross output of the } i^{\text{th}} \text{ industry} \end{array}$$

$$y = \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_i \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \quad \begin{array}{l} = \text{an } n \times 1 \text{ final demand vector} \\ \text{where } y_i = \text{final demand for } i^{\text{th}} \text{ industry's output} \end{array}$$

$$X = \begin{bmatrix} x_{11} & \dots & x_{1j} & \dots & x_{1n} \\ \vdots & & \vdots & & \vdots \\ x_{i1} & \dots & x_{ij} & \dots & x_{in} \\ \vdots & & \vdots & & \vdots \\ x_{n1} & \dots & x_{nj} & \dots & x_{nn} \end{bmatrix} = \text{an } n \times n \text{ interindustry transactions matrix}$$

where  $x_{ij}$  = amount of output  $i$  purchased by industry  $j$  for use as input.

= intermediate demand for output  $i$  by industry  $j$ .

Now the  $i^{\text{th}}$  gross output is the sum of the  $i^{\text{th}}$  industry intermediate sales to the  $n$  industries plus the final demand for the output of industry  $i$ , shown as equation (1):

$$(1) \quad x_i = \sum_{j=1}^n x_{ij} + y_i \quad i = 1, \dots, n.$$

Total intermediate demand,  $\sum_{j=1}^n x_{ij}$ , is the horizontal sum of a row of  $X$ , and  $y_i$  is final demand for the output of industry  $i$ .

Let

$$(2) \quad a_{ij} = \frac{x_{ij}}{x_j} = \text{technical or direct coefficient, i.e., input from industry } i \text{ per unit of output of industry } j$$

The  $n \times n$  matrix  $A$  whose elements are the technical coefficients  $a_{ij}$ , is

$$(3) \quad A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}$$

The  $a_{ij}$ 's implicitly depend upon the existing production technology assumed to be of the fixed coefficient type.

Now multiplying both sides of (2) by  $x_j$  and summing over  $j$  yields

$$\sum_{j=1}^n x_{ij} = \sum_{j=1}^n a_{ij} x_j$$

which can be substituted into (1):

$$(4) \quad x_i = \sum_{j=1}^n a_{ij} x_j + y_i \quad i = 1, \dots, n.$$

This system of equations may be written more compactly in matrix form for all industries as

$$(5) \quad x = Ax + y$$

The direct and secondary or indirect output requirements may be obtained by the following matrix operations:

$$x - AX = y$$

$$(I-A)x = y \quad \text{where } I = n \times n \text{ identity matrix}$$

If  $(I-A)$  satisfies the Hawkins-Simons conditions, then its inverse  $(I-A)^{-1}$  exists and thus the final demand and gross output vectors are non-negative.<sup>3</sup>

$$(6) \quad x = By$$

where

$$B = (I-A)^{-1} = \begin{bmatrix} b_{11} & \dots & b_{1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ b_{n1} & \dots & b_{nn} \end{bmatrix}$$

is a  $n \times n$  Leontief inverse matrix of direct and indirect outputs required as inputs per unit of final demand. The coefficient  $b_{ij}$  equals total (direct plus indirect) required increased output from industry  $i$  when final demand for industry  $j$ 's output increases by one unit. Equation (6) is the familiar relationship which is useful for planning and forecasting purposes. Given final demand as the independent variable, one can calculate the dependent variable in the form of gross industry output required to satisfy that level of final demand. Given forecasts of  $y$ , say  $y^*$  and  $B$ , it is a simple matter to find the product  $By^* = x^* =$  forecast of output.

In general the current practice of forecasting industrial or sectoral output with (6) relies on ad hoc procedures to generate the necessary final demand forecasts. We propose to "drive" the Ohio I/O model for forecasting purposes by the final demand vector projected from the OEM. That is, an econometric model will provide the forecast of  $y$  which can be used in turn to forecast gross output,  $x$ . A well-specified regional aggregate demand model should provide a better conceptual framework for forecasting the final vector than ad hoc procedures.

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<sup>3</sup>The Hawkins-Simons conditions, which are equivalent to the strong and weak solvability conditions, insure that  $(I-A)^{-1}$  exists and guarantee non-negativity of these two vectors, thereby assuring their meaning in an economic sense. See Nikaido [13, pp. 90-94].

## The Ohio Input-Output Model

This section is about the non-survey construction of the Ohio I/O model. After a brief presentation of the analytics of the non-survey technique, we discuss problems encountered in constructing the required data, testing the I/O model for consistency and reliability, and using the model for forecasting purposes.

The use of an I/O model for forecasting purposes assumes a knowledge of B, the Leontief-inverse matrix, which in turn requires a knowledge of production requirements, i.e., the direct coefficients matrix, A. In order to derive the Ohio I/O direct coefficient matrix and establish the linkages between the Ohio I/O and the OEM, two simplifying steps were taken. First the Ohio economy was aggregated to six private sectors; and an Ohio I/O model was built using a non-survey procedure for these sectors. (Later in this section we discuss a 77-sector I/O model.) Aggregation involved a simple summation of transactions and total gross outputs along the following lines:

<u>Section #</u>	<u>Ohio Economic Sectors</u>	<u>Corresponding U.S. I/O Industry Number(s)</u>
1	Agriculture, mining, transportation, communications, utilities	1-10, 65-68
2	Construction	11-12
3	Manufacturing	13-64
4	Trade	69
5	Finance, insurance & real estate	70-71
6	Services and other	72-77

The reason for aggregating the 77 private I/O U.S. industries to the six sectors shown above was to make the industry classifications of the Ohio I/O model consistent with those of the OEM. The industry classifications of the OEM are based on the Standard Industrial Classification (SIC) code [3]. The advantage of working with this degree of aggregation<sup>4</sup> is the simplicity with which the consistency of one model could be checked against the other.<sup>5</sup>

<sup>4</sup>Two studies, Doeksen and Little [2] and Williamson [24] suggest that severe unweighted aggregation has little impact upon sectoral multipliers and hence on impact analysis. However, Hewings [6] points out that unless two theorems are satisfied, aggregation bias could distort the end results. He demonstrates how the violation of these two theorems in the case of the aggregation of the 1963 state of Washington input-output model affects the level of gross output. One of these two theorems applies to our aggregation. It states that if the final demands are proportional to those of the base period, then the aggregation bias is zero. The final demands of our aggregation of the U. S. I/O model are not proportional to those of the base period; however, they are not distinctly different from the base period.

<sup>5</sup>We did not include the 77-sector version of the Ohio I/O model in this paper because of space considerations--the table of direct coefficients alone fills 26 computer print-out pages.

## The Non-Survey Method

The second simplifying step was to use an existing computer algorithm of a non-survey technique to derive the Ohio direct coefficients from the U. S. direct coefficients.<sup>6</sup> This algorithm requires the U. S. transactions matrix estimates of Ohio final demands and estimates of Ohio total gross outputs. The Bureau of Labor Statistics of the U. S. Department of Labor provided the U. S. transactions matrix for 1970. Ohio final demands for 1970 were obtained from the latest version of the OEM [7, 8].

The algorithm is quite straightforward.<sup>7</sup> All variables are for the same year, 1970.

Let  $x_j$  = Ohio total gross input for the  $j^{\text{th}}$  sector

$y_i$  = Ohio final demand for the  $i^{\text{th}}$  sector

$a_{ij}^{\text{US}}$  = U. S. direct coefficient

$a_{ij}^{\text{o}}$  = Ohio direct coefficient

Assume for the moment that

$$a_{ij}^{\text{o}} = a_{ij}^{\text{US}}$$

The  $i^{\text{th}}$  Ohio total gross output required to satisfy tentative intermediate demand and final demand is then

$$x_i^{\text{T}} = \sum_{j=1}^n a_{ij}^{\text{US}} x_j + y_i \quad i = 1, \dots, n$$

The tentative Ohio total gross output required ( $x_i^{\text{T}}$ ) or demanded of industry  $i$  and the actual Ohio total gross output ( $x_i$ ) supplied by  $i$  are compared as follows:

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<sup>6</sup>This computer algorithm was originally developed by C. J. Palmer and M. R. Layton [14] at the U. S. Department of Agriculture, Economic Research Service, East Lansing, Michigan. It was subsequently translated and modified for use on The Ohio State University computer by the Center for Human Resources Research, The Ohio State University.

<sup>7</sup>This algorithm can be classified as a variant of the regional commodity balances approach and as such is very similar to the "Supply-Demand Pool" (SDP) non-survey method reviewed by Schaffer and Chu [19] and Richardson [17]. It is easily shown that the adjustment criteria of the SDP method and the algorithm herein are identical.

$$\frac{x_i}{x_i^T} = D_i \quad i = 1, \dots, n$$

The decision rule for adjusting the  $a_{ij}^{US}$  is formulated for each  $i = 1, \dots, n$ .

$$(i) \text{ if } D_i \geq 1 \text{ then } a_{ij}^o = a_{ij}^{US} \quad j = 1, \dots, n$$

$$(II) \text{ if } D_i < 1 \text{ then } a_{ij}^o = D_i a_{ij}^{US} \quad j = 1, \dots, n$$

Interregional trade flows (not calculated in this paper) are derived from a calculated final demand ( $y_i^C$ ) based on the  $a_{ij}^o$ :

$$y_i^C = x_i - \sum_{j=1}^n a_{ij}^o x_j \quad i = 1, \dots, n$$

Then exports ( $e_i$ ) from the  $i^{\text{th}}$  Ohio industry are:

$e_i = y_i^C - y_i$  where  $i = 1, \dots, n$  and  $y_i$  = actual final demand and imports ( $m_{ij}$ ) from the  $i^{\text{th}}$  U. S. sector to the  $j^{\text{th}}$  Ohio sector are:

$$m_{ij} = a_{ij}^{US} x_i - a_{ij}^o x_i \quad i, j = 1, \dots, n$$

The imports can be expressed as a matrix

$$M = [m_{ij}] \quad i, j = 1, \dots, n$$

Total imports of Ohio's  $j^{\text{th}}$  industry are

$$S_j = \sum_{i=1}^n m_{ij} \quad j = 1, \dots, n$$

The calculation of trade flows is the weakest link in all non-survey procedures.<sup>8</sup> As discussed by Schaffer and Chu [19], the SDP method assumes a considerable degree of regional self-sufficiency. In other words, as much of the total gross output as possible is assumed to be used within the region to satisfy the regional demand. This assumption is mitigated by the fact that the trade flows, which can be calculated as described above, were not used to project employment. Furthermore, the use to which we put the conjoined Ohio I/O econometric model, namely estimating employment by sector, reflects the contribution of Ohio net exports as part of the estimate of  $y$ .

<sup>8</sup> Schaffer and Chu [19] offer the best empirical verification of this fact. They compare the actual trade flows from the survey-based 1963 Washington I/O table with trade flows calculated from various non-survey techniques.

## Projecting the 1970 Total Gross Outputs (TGO's) for Ohio

In order to find the  $a_{ij}^0$ 's, the 1970 total gross outputs for Ohio are required. However, because no data were available for 1970, a method was devised to project the 1970 Ohio's TGO's using Ohio employment data, productivity rates, and 1963 output data.<sup>9</sup> Briefly, this method entails projecting the 1963 direct requirements for labor using average annual productivity growth rates. From the projected 1970 direct requirements and knowledge of the actual 1970 employment, the 1970 TGO's can be estimated according to the following steps:

1. Aggregate  $x_i^{63}$ ,  $i = 1, \dots, 77$  for 1963 to 6 sectors to get  $x_k^{63}$ ,  $k = 1, \dots, 6$ .

Similarly for employment: aggregate  $\epsilon_i^{63}$ ,  $i = 1, \dots, 77$  to obtain  $\epsilon_k^{63}$ ,  $k = 1, \dots, 6$ , where  $\epsilon_i$  is level of employment in the  $i$ th industry.

From now on, we will use the subscript  $i$  to indicate one of the (77) disaggregate industries and use the subscript  $k$  to indicate an aggregate industry.

2. Calculate 1963 direct requirements for labor:

$$l_k^{63} = \frac{\epsilon_k^{63}}{x_k^{63}}, \quad k = 1, \dots, 6$$

3. Obtain average annual estimates of productivity growth  $p_k$  (percent growth rate)  $k = 1, \dots, 6$ . Calculate the growth of productivity from 1963 to 1970 =  $(1 + p_k)^7$ .

4. Calculate 1970 labor productivity, the inverse of the direct requirement:

$$(l_k^{63})^{-1} (1 + p_k)^7 = (l_k^{70})^{-1} = \left( \frac{\epsilon_k^{70}}{x_k^{70}} \right)^{-1}$$

5. Find 1970 TGO's (aggregate):

$$(l_k^{70})^{-1} \cdot \epsilon_k^{70} = x_k^{70} \quad k = 1, \dots, 6$$

Data and calculations up to this point are included in Appendix I.

6. To disaggregate  $x_k^{70}$  into 77 1/0 industries, we derived the weights as follows: Assume that the TGO mix within each aggregate sector remains constant over the period 1963-1970. Specify the initial

<sup>9</sup>Disaggregate data for 1963 are readily available by the 77-sector classification in Polenske [15], Rodgers [18], and Scheppach [20].



set of TGO weights as follows:

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$$\frac{x_h^{63}}{x_k^{63}} \quad (\text{where } k \text{ is the } k^{\text{th}} \text{ aggregate sector TGO where } k = 1, \dots, 6; \text{ the numerator denotes the } h^{\text{th}} \text{ I/O industry TGO within the } k^{\text{th}} \text{ aggregate sector}).$$

There are, however, several limitations to step #6. We have made productivity growth projections only for the aggregate industries. Clearly it would be more desirable to project each  $x_k^{63}$  ( $k = 1, \dots, 77$ ) separately rather than  $x_k^{63}$  ( $k = 1, \dots, 6$ ). This is likely to produce more accurate projections because productivity for each  $i^{\text{th}}$  I/O industry could be used. In this case, because the disaggregation weighting scheme would not have to be used a possible source of aggregation error is likely to be reduced or eliminated.

### Disaggregating Final Demands

As mentioned above, the 6-sector final demand estimates ( $y_k^{\text{OEM}}$ ) from the OEM are classified on an SIC basis. Therefore, to be used in the algorithm for the 77-sector version of the Ohio I/O model, the 6 final demands must be disaggregated to 77. Two possible methods were considered:

1. Disaggregate by employment weights  $\frac{\epsilon_i^{63}}{\epsilon_k^{63}}$ . However, because this method assumes  $\frac{\epsilon_i}{x_i}$  is constant, over time, it is implied that  $\frac{\epsilon_i}{y_i}$  will also be constant over time. This is less likely to hold because of the variation in the mix of intermediate demands.
2. To circumvent the need for this assumption, we will use weights based on 1970 projections of final demand for all I/O categories made by Scheppach [20] and symbolized as  $y_i^S$ . These projections were made from 1963 data using various ad hoc procedures (e.g., population projections). We assume the product mix of 1970 final demand as estimated by Scheppach is correct. Scheppach's categories of final demand differ from the OEM's, but the two total estimates of Ohio final demand are reasonably close:

1970 Final Demand Estimates  
(Billions of 1958 Dollars)

	(1) Total GSP-Ohio (all sectors)	(2) GSP-Ohio (six sectors)
OEM	39.9	36.5
Scheppach	36.4	32.0
Difference	3.5	4.3
Actual value (L'Esperance [9])	40.1	-

The difference is within 10 percent of the OEM final demand for all sectors, and within 12 percent for the six sectors. Therefore, we imposed the product mix from Scheppach on the OEM final demand in the following procedure:

- (1) Aggregate  $y_i^s$  to 6 sectors,  $y_k^s$ , where  $i = 1, \dots, 77$ ,  $k = 1, \dots, 6$ , by simple summation.
- (2) Calculate weights  $\frac{y_i^s}{y_k^s}$
- (3) Disaggregate  $y_k^{OEM}$  :

$$\frac{y_i^s}{y_k^s} \cdot y_k^{OEM} = y_i^{OEM} \quad \begin{array}{l} i = 1, \dots, 77 \\ k = 1, \dots, 6 \end{array}$$

### Estimating Total Employment for Each Sector

In the previous two sections, we have needed estimates of total employment for the 6 aggregate industries as well as the 77 disaggregate industries. Data on the latter are available from only one source for 1963, Rodgers [18]. Data are available for the 6 industries from several sources. For instance, Rodgers' data [18] can be aggregated to 6 sectors for 1963 employment figures. The Ohio Bureau of Employment Services (OBES) has alternative data, as shown in the table below, most of which are within the same range as the I/O data. The major exception is Sector #1 for which the OBES figures are about 150 percent of the I/O figures because of the agricultural component. To confuse matters further, if U. S. Department of Agriculture (USDA) statistics are used, as shown below, instead of the OBES estimates of agricultural employment, the discrepancy is even greater.<sup>10</sup>

1963 Employment  
(thousands)

Sector Number	(Rodgers (Aggregated I/O)	OBES (Wage & Salaried Only)	OBES & USDA
1	251.6	384.0	470.8
2	149.7	130.6	
3	1188.4	1234.5	
4	598.8	611.1	
5	123.2	126.5	
6	393.8	394.4	

<sup>10</sup>This difference is primarily due to collection techniques and definitions of agricultural employment. From our standpoint, the USDA double counts some workers who engage in several occupations. We, therefore, did not use USDA estimates.

In Section III.2 we used aggregate 1963 I/O employment data to calculate the direct coefficients of labor ( $\ell_k^{63}$ ) because we assumed that Rodger's estimates of employment are consistent with total gross output. Thus, any errors in either employment or total gross output should cancel out in  $\ell_k^{63}$ . However, when we converted  $\ell_k^{70}$  to  $x_k^{70}$  via  $\epsilon_k^{70}$ , the data we used were OBES wage and salaried employment. (Rodger's data do not extend to 1970.) This figure did not include self-employed, domestic and unpaid family workers. The OBES did not disaggregate this category by industry, so we used the following ad hoc procedure to distribute the self-employed, domestic and unpaid family workers (all "self-employed" for short) across industries:

1. Assume the percentage of self-employed is constant over sectors and the percentage is equal to the overall percentage of

$$\frac{\text{Self-employed non-agricultural workers}}{\text{Non-agricultural wage \& salary workers}} = 9.5 \text{ percent for 1970}$$

2.  $.095_k$  = self-employment in the  $k^{\text{th}}$  sector where  $k$  is wage and salary employment only.
3. For total employment in each sector,  $\epsilon_k = k + .095k$

Sector Number	k	+	.095 <sub>k</sub>	=	$\epsilon_k^{70}$
1	349.4*		33.1		382.5
2	156.8		14.9		171.7
3	1407.4		133.7		1541.1
4	759.9		72.2		832.1
5	151.4		14.4		165.8
6	546.1		41.9		598.0

\*OBES figures: 1970 farm labor 103.3 thousand

### The Six Sector Aggregate Model

Table 1 presents the calculated 1970 direct coefficients for Ohio. From these, Table 2 was derived as the Leontief inverse, or the direct and indirect coefficients. Also, Table 1 combined with the calculated TGO's (see Section III.2 and Appendix II) yields Table 3, the 1970 Ohio transactions matrix in millions of 1958 dollars.

### Forecasting Employment by Industry in Ohio

This section discusses the procedure and some of the problems for project employment by industry for Ohio. The Ohio Leontief inverse matrix, B, must accurately describe the Ohio economy, and be relatively stable over time. In addition, the Ohio econometric model must accurately estimate Ohio's final

TABLE 1: Ohio 1970 Input-Output Table - 6 Sectors, Direct Purchases per One Dollar of Output

	1	2	3	4	5	6
Industry Purchasing Industry Producing	Agriculture Mining & Transportation	Construction	Manufacturing	Wholesale & Retail Trade	Finance Insurance & Real Estate	Services & Miscellaneous
1 Agriculture, Mining & Transportation	0.1561	0.0406	0.1018	0.0336	0.0330	0.0505
2 Construction	0.0176	0.0001	0.0016	0.0065	0.0547	0.0102
3 Manufacturing	0.0843	0.4017	0.3989	0.0488	0.0214	0.1856
4 Wholesale & Retail Trade	0.0234	0.0857	0.0320	0.0154	0.0140	0.0264
5 Finance, Insurance & Real Estate	0.0467	0.0109	0.0140	0.0696	0.1212	0.0638
6 Services and Miscellaneous	0.0307	0.0545	0.0289	0.0749	0.0398	0.0633

Note: 1. Each entry shows the total dollar production directly and indirectly required from the industry at the left of the table per dollar of deliveries to final demand by the industry at the top.

2. The entries in this table were calculated using final demands from the Ohio Econometric Model and total gross output projected by the productivity trend method.

TABLE 2: Ohio 1970 Input-Output Table - 6 Sectors, Direct and Indirect Requirements per One Dollar Delivery to Final Demand

	1	2	3	4	5	6
Industry Purchasing Industry Producing	Agriculture, Mining & Transportation	Construction	Manufacturing	Wholesale & Retail Trade	Finance, Insurance & Real Estate	Services & Miscellaneous
1 Agriculture, Mining & Transportation	1.2196	0.1499	0.2178	0.0671	0.0669	0.1169
2 Construction	0.0267	1.0083	0.0104	0.0142	0.0651	0.0193
3 Manufacturing	0.2118	0.7364	1.7293	0.1344	0.1150	0.3736
4 Wholesale & Retail Trade	0.0407	0.1186	0.0648	1.0267	0.0289	0.0472
5 Finance, Insurance & Real Estate	0.0757	0.0487	0.0493	0.0939	1.1508	0.0951
6 Services and Miscellaneous	0.0545	0.0978	0.0683	0.0932	0.0608	1.0919
Output Multiplier	1.6291	2.1598	2.1399	1.4295	1.4876	1.7439

Note: 1. Each entry shows the total dollar production directly and indirectly required from the industry at the left of the table per dollar of deliveries to final demand by the industry at the top.  
 2. The entries in this table were calculated using final demands from the Ohio Econometric Model and total gross output projected by the productivity trend method.

TABLE 3: Ohio 1970 Input-Output Table - 6 Sectors, Interindustry Transactions  
(in millions of 1958 dollars)

	1	2	3	4	5	6
Industry Purchasing Industry Producing	Agriculture, Mining & Transportation	Construction	Manufacturing	Wholesale & Retail Trade	Finance, Insurance & Real Estate	Services & Miscellaneous
1 Agriculture, Mining & Transportation	2159.5	217.0	5424.3	340.3	270.7	359.1
2 Construction	243.0	0.4	85.1	65.9	448.2	72.4
3 Manufacturing	1167.0	2144.9	21245.4	494.0	175.4	1319.7
4 Wholesale & Retail Trade	323.7	457.8	1705.1	156.3	114.3	187.4
5 Finance, Insurance & Real Estate	646.0	58.2	744.4	704.1	992.8	452.1
6 Services and Miscellaneous	424.7	291.1	1537.8	757.8	326.3	450.1

Note: 1. The entries in this table were calculated using final demands from the Ohio Econometric Model and total gross output projected by the productivity trend method.

demand vector  $y$ . If these conditions hold, then post multiplying  $B$  by any  $y$  in year  $t$  yields the projected total gross output vector  $x$  in year  $t$ . Transforming this vector of projected industry outputs into employment requires relating industry employment to industry output in year  $t$ .

To handle this problem we assume that employment in each sector  $\epsilon_i$  is proportional to the corresponding sector's output  $x_i$ . A similar approach is discussed by Miernyk [11]. This means that employment is functionally related to output as follows:

$$\epsilon_i^t = \lambda_i(t)x_i^t$$

The inverse of  $\lambda_i(t)$  is the often used ratio  $x_i^t/\epsilon_i^t$  which has been described in many studies as labor productivity. It should be noted that most if not all sectors are characterized by increasing productivity and thus  $\lambda_i(t)$  is a decreasing function of time, e.g.,  $\frac{d\lambda_i(t)}{dt} < 0$ . This measure, however, is potentially misleading since the decline of  $\lambda_i(t)$  over time may reflect a number of factors other than improvements in labor efficiency. Increasing capital intensity, managerial efficiency, better utilization of capital, more external economies in addition to improvements in the quality of labor may account for the decline in  $\lambda_i(t)$ .

Direct and indirect labor requirements per one dollar of final demand are calculated by first diagonalizing the  $n \times 1$  vector  $\lambda$  into an  $n \times n$  matrix  $L_t$ . The resulting matrix is then post multiplied by the Ohio Leontief inverse matrix, and the elements of the matrix  $L_t B$  represent the total direct and indirect employment requirements of industry  $i$  needed to provide for one unit of final demand in industry  $j$ . This resulting matrix is then post multiplied by the projected final demand vector to arrive at the direct and indirect employment requirement estimates for each sector to meet the projected final demand mix. Projected employment by sector may be expressed as follows:

$$L_t B y_t = L_t x_t = \epsilon_t$$

In expanded form this is:

$$\begin{bmatrix} \lambda_1(t) & & & & 0 \\ & \lambda_2(t) & & & \\ & & \cdot & & \\ 0 & & & \cdot & \\ & & & & \lambda_n(t) \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ b_{n1} & b_{n2} & \dots & b_{nn} \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ \cdot \\ \cdot \\ y_n(t) \end{bmatrix} =$$

$$\begin{bmatrix} \ell_1(t) & & & & 0 \\ & \ell_2(t) & & & \\ & & \ddots & & \\ 0 & & & & \ell_n(t) \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} = \begin{bmatrix} \ell_1(t) & x_1(t) \\ \ell_2(t) & x_2(t) \\ \vdots & \vdots \\ \ell_n(t) & x_n(t) \end{bmatrix} = \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \\ \vdots \\ \varepsilon_n(t) \end{bmatrix}$$

where  $\varepsilon_i(t)$  is projected employment for industry  $i$  in year  $t$ .

In Table 4 are presented estimates of employment produced by the conjoined I/O econometric model for selected years. They are broken down by the six major sectors. Except for three cases (1965 and 1966 in services and miscellaneous and 1965 in agriculture, etc.) all percentage errors are less than ten percent and many are less than three percent. For 1970 it is expected that the percentage error be zero; and it is zero if the percentage error is rounded off to within 1/10 of one percent. Except for manufacturing the percentage errors for 1969 and 1971 are low, reflecting reliability of the Ohio I/O model, which is a 1970 model, for these years. For the years 1965-1967 the forecast error increases, reflecting the inadequacy of the 1970 Ohio I/O model for these years. This suggests that revising the Ohio I/O model annually in conjunction with the yearly updating of the OEM would be most appropriate for producing consistently reliable forecasts.

It should be noted that the employment projections of this model will be eventually compared with forecasts of employment from the OEM to provide alternate estimates of employment by industry. If consistency in the forecasts can be achieved between the two sets of forecasts, then we can use the more disaggregated I/O model to make disaggregated industrial employment projections. These estimates of employment by industry may be useful when combined with occupation by industry matrices for transforming the industrial employment projections into occupational employment projections (discussed in the next section). Such projections are of great value to local manpower planners, vocational guidance counselors and education planners. Furthermore, this procedure may also be expanded to cover a sub-state area if the appropriate data are available to forecast the region's final demand.

The requirements for using an I/O model for forecasting total regional output have been given elsewhere Richardson [17] and will not be considered here. It should be mentioned, however, that our procedure for estimating employment outlines all of the necessary steps for projecting total output.

Our procedure involves making the assumption of a proportionate relationship between employment and output. The inability to deduce the correct relationship between industry employment and industry output will, of course, limit the usefulness of this model in forecasting industry employment. It is possible that the relationship can be more accurately estimated by regression techniques which adequately estimate the relationship between employment and output. Such a relation would involve testing other functional specifications than the one of strict proportionality, and would incorporate additional explanatory variables.



TABLE 4: Estimates of Ohio Employment by Major Sector  
(Thousands of Workers)

Year	Agriculture, Mining and Transportation			Construction		
	Estimated	Actual	% Error	Estimated	Actual	% Error
1965	309.0	354.7	- 12.88	187.0	186.1	+ .48
1966	322.6	348.3	- 7.38	203.6	198.1	+ 2.78
1967	322.0	347.6	- 7.36	195.3	201.2	- 2.93
1968	344.3	352.8	- 2.41	212.6	215.0	- 1.12
1969	346.9	354.2	- 2.06	220.1	226.0	- 2.61
1970	355.5	355.6	- .03	204.5	204.6	- .05
1971	365.4	354.0	+ 3.22	195.8	195.6	+ .10
	Manufacturing			Wholesale and Retail Trade		
1965	1411.6	1360.6	+ 3.75	667.3	654.9	+ 1.89
1966	1392.8	1440.9	- 3.34	701.6	683.0	+ 2.72
1967	1356.9	1438.4	- 5.67	711.2	705.6	+ 0.79
1968	1460.4	1471.3	- 7.40	750.6	735.6	+ 2.04
1969	1369.6	1509.4	- 9.26	771.0	768.0	+ 0.39
1970	1447.6	1447.9	- .02	781.8	781.9	- 0.01
1971	1493.8	1370.6	+ 8.99	794.1	791.3	+ 0.35
	Finance, Insurance, Real Estate			Services and Miscellaneous		
1965	144.0	134.2	+ 7.30	848.1	725.2	+ 16.95
1966	149.3	137.2	+ 8.82	883.3	652.3	+ 35.41
1967	153.8	143.1	+ 7.48	871.5	812.4	+ 7.27
1968	159.2	149.3	+ 6.63	907.8	855.7	+ 6.09
1969	158.9	154.5	+ 2.85	918.2	898.8	+ 2.16
1970	161.6	161.6	0.00	941.3	941.6	- 0.03
1971	165.7	165.6	+ 0.12	953.9	972.3	- 1.89

Because of the importance of the indirect employment creating effects of industrial expansion, it is useful to calculate employment multipliers for each industry. These multipliers reflect the ratio of direct and indirect employment to direct employment requirements from all sectors associated with an industry providing one additional unit of output to final demand. Thus a greater employment multiplier associated with industry  $i$  as compared to industry  $j$  implies that the indirect employment, when compared with direct employment generated by industry  $i$ , is greater than industry  $j$ .

Industry  $i$ 's employment multiplier is calculated by summing the  $j^{\text{th}}$  column of the matrix  $L_t B$  and dividing the resulting total by  $\ell_j$ . Thus

$$\text{Employment multiplier} = \beta_j / \ell_j$$

$$\text{where } \beta_j = \sum_{i=1}^n \ell_i b_{ij}$$

### Converting Projected Employment by Industry into Employment by Occupation

The above model projects employment by industry for the private sector. Forecasts of employment in the public sector can be given by the econometric model or ad hoc procedures. Using census data, the public sector employment can be distributed across industries by functional categories. The purpose of this distribution is to obtain forecasts of employment by industry which are consistent with the new SIC industrial classification system as discussed in Appendix II, and also with the occupation by industry matrices available from the Bureau of Labor Statistics [22]. Combining the forecasts of employment by industry with the occupation by industry matrix permits the forecasting of employment requirements by occupation for the region. Such estimates are useful for regional manpower planners attempting to allocate scarce funds among alternative educational programs as well as providing information for vocational decisions to new entrants into the region's work force.

Briefly, the procedure is to compute the region's occupation by industry matrix from the most recent census data and to modify the cells for the projected year by using estimates of the Bureau of Labor Statistics of the changes in the corresponding cells of the national occupation by industry matrix [23]. The resulting occupation by industry matrix for the projected year  $t$  is  $N^t = (n_{ij}^t)$  ( $i = 1, \dots, m, j = 1, \dots, r$ ) where  $n_{ij}^t$  represents the proportion of occupation  $i$  in industry  $j$ 's work force in year  $t$ . This matrix  $N^t$  is then post multiplied by the  $m \times 1$  vector of projected employment by industry,  $\epsilon^t$ , to arrive at an  $m \times 1$  vector of projected employment by occupation in year  $t$ ,  $\epsilon^{*t}$ .

Symbolically, the operation is as follows:

$$N^t \epsilon^t = \epsilon^{*t}$$

where

$$N^t = \begin{bmatrix} n_{11}^t & n_{12}^t & \dots & n_{1r}^t \\ \vdots & \vdots & & \vdots \\ n_{m1}^t & n_{m2}^t & & n_{mr}^t \end{bmatrix}$$

$$e^t = \begin{bmatrix} \epsilon_1^t \\ \vdots \\ \epsilon_m^t \end{bmatrix}$$

and

$$\epsilon^{*t} = \begin{bmatrix} \epsilon_1^{*t} \\ \vdots \\ \epsilon_m^{*t} \end{bmatrix}$$

### Conclusion

In this paper we have conjoined a theory of production and a theory of aggregate demand in the form of an integrated Ohio I/O econometric model. The end result of this effort was to estimate employment by industry, using the OEM to "drive" the final demand sector of the I/O model. While this was done at a highly aggregative level, it was pointed out how a disaggregated 77 sector Ohio I/O model could be used with a more aggregative econometric model. Finally, a procedure was outlined for converting employment estimates into employment by occupation.

An exercise of this kind is the natural outgrowth of the development of different types of empirical macroeconomic models which must ultimately relate to one another in acknowledging the interdependence of economic behavior. In forcing this issue at the regional level we hope to stimulate discussion, interest and research in balancing the efforts poured into building I/O models with the growing activity in regional econometric model building.

APPENDIX I: Calculation of Total Gross Output

k	$P_k$	$(1+P_k)^7$	$\ell_i^{63} = \frac{\epsilon_k^{63}}{x_k}$	$(\ell_k^{63})^{-1}$	$(1+P_k)^7 \cdot (\ell_k^{63})^{-1}$	$\epsilon_k^{70c}$ (Thousands of Employees)	$x_k^{70} =$ $(\ell_k^{70})^{-1} \cdot \epsilon_k^{70}$ (Millions of 1963 \$)
					$= (\ell_R^{70})^{-1}$		
1	.05	1.407	.039	25.703	36.166	382.593 <sup>b</sup>	13,836.898
2	.016	1.116	.036	27.829	31.100	171.696	5,339.710
3	.032	1.247	.036	27.721	34.559	1541.103	53,259.554
4	.033	1.255	.103	9.689	12.161	832.091	10,118.859
5	.020	1.149	.023	43.012	49.406	165.783	8,190.928
6	.015	1.110	.093	10.716	11.893	597.980	7,111.773

Note: See Section III.2 for explanation of symbols.

<sup>a</sup>Employment data from the aggregation of Rodgers [18] 77 sector data (see discussion in Section III.4).  
Total gross output also estimated by Rodgers [18].

<sup>b</sup>BLS (Bureau of Labor Statistics) figures only: 1970 Farm Labor = 103.3 thousand workers.  
<sup>c</sup>1970 employment data from the Ohio Bureau of Employment Services (OBES).

APPENDIX 11. A Note on Input-Output (I/O) Industrial Classifications as Formulated by the Bureau of Economic Analysis (BEA) U. S. Department of Commerce

This note is intended to clarify some problems in the classification description given by Rodgers [18] for identifying the content of selected I/O sectors and comparing them with other classification systems. The sector descriptions relating I/O industry numbers and SIC codes are given in Rodgers in Table A-1 and relate to the 1957 SIC classification system. This system has been changed in the latest (1972) SIC manual [3] so that "all establishments primarily engaged in the same type of economic activity are classified in the same four-digit industry regardless of their types of ownership" [3, p. 11]. Thus the government establishments are classified by their primary economic activity, rather than by the type of owner. For example, a public high school teacher is classified in SIC 82 (education) and not in SIC 92 (government) as before.

The descriptions given by Rodgers for I/O-77, 78, 79 and 84 indicate that the bulk of government sector employment and income occurs in I/O-84, although the description for I/O-77, "medical, educational services, and nonprofit organizations," when viewed without the description of I/O-84, "government industry," may be interpreted to include income and employment generated from government medical and educational activities. To enable a clearer understanding of the content of these sectors we give descriptions, as given by Rodgers, for I/O-77, 78, 79 and 84.

I/O-77 Medical, Educational Services and Nonprofit Organizations: This industry is defined on a modified activity basis and includes the total value of the covered services wherever they occur..." [3, p. 88].

I/O-78 Federal Government Enterprises: "This industry covers the activities of those Federal Government agencies, with separate accounting records, that cover over one-half of their current operating costs by the sale of goods and services to the general public.

Output of Federal enterprises that are comparable to those of private establishments are transferred to the appropriate private industries for distribution. Outputs for which there are no private counterparts are distributed directly to consuming industries" [3, p. 91].

I/O-79 State and Local Government Enterprises: "This industry covers the activities of the state and local government agencies, with separate accounting records, that cover over one-half of their current operating costs by the sale of goods and services to the general public. State and local government enterprises include: (1) gas and electric utilities; (2) water supply facilities; (3) transit facilities; (4) liquor stores; (5) water transportation and terminals; (6) air transportation facilities; (7) highway toll facilities, and such activities as (8) sewers and sewage disposal; (9) low-cost housing and urban renewal; and (10) some miscellaneous activities such as off street parking and city markets. The outputs of most

state and local government enterprises are transferred to private industries in which the comparable activity is performed. Outputs for which there are no private counterparts were distributed directly to consuming industries" [3, pp. 92, 93].

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