SHORT-RUN DISEQUILIBRIA IN URBAN SPATIAL STRUCTURES*

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Shocks of Urban Settings

The basic assumptions of this paper are that first, there exists a state along the evolution of urban areas that is characterized by a period of stable exogenous conditions (environment) and stable technology during which the urban economy can experience long-run equilibrium; second, at some point in time, and due to either external disturbances in the environment or changes in the technology of production, the interrelationships between the rest of the world and the urban setting change. A possible result of such changes may be in- or out-migration of labor as a response to a wage or utility differential equalization process.

Such exogenous shocks, if unique in their occurrences, are not anticipated; thus, no expectations are formed by the producers or consumers of urban commodities, and there is no reason for speculation by landowners on the suburban or adjacent agricultural land. However, in the case of successive shocks, occurring at random (or stochastic) intervals over a period of time, expectations and speculation on land are built up as a response to the expected magnitude of such disturbances (assumed to be manifested here in the form of in-or out-migration of labor) and their time sequence. The intensity of a succession of shock waves may or may not remain constant in the general case. The length of time among shocks and their magnitude may be related, since simple observations suggest that over longer intervals the regularity in the magnitude of the shocks is higher than in the case of smaller intervals. Thus, such a stochastic or random occurrence of shocks would be an important factor in establishing "learning" in the behavior of the urban landowners, as well as in the behavior of old urban residents. Such a probabilistic approach to urban shocks is essential in examining the efficiency of speculation on land and the existence or lack of myopia in the utility maximizing behavior of urban households over time and space.

Another type of shock is the endogenous type, i.e., the new household formation (and household dissolution) as the result of natural population increases and other social forces. This endogenous change may or may not exhibit the characteristics of randomness mentioned for the case of exogenous shocks; their impact is however combined for the build-up of expectations among the economic agents.

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However, this paper is not to deal with such an ambitious probabilistic theory, but rather with the study of the short-run effect of a unique shock. Obviously, the urban economies are shocked (as already stated) continuously at varying scales of intensity in various parameters, and here we will concentrate on the case of a unique change occurring in the supply of the labor force.

The fact that the short-run effects of the sudden influx of the labor force are to be studied implies that the urban economy will be observed during a state of disequilibrium in certain markets with high adjustment costs (residential sector) or low convertibility due to indivisibilities (transportatio sector). We will examine the state of the city "the day after the shock" because it is acknowledged here that contemporary cities in developed countries live a succession of such "day after" conditions, i.e., they experience a succession of short-run disequilibria resulting in a continuum of inefficient allocation schemes.

The other extreme, the case of an urban economy experiencing a succession of endogenous regular shocks (exponential growth of the labor force) as a succession of long-run equilibria, was studied by Dendrinos [2], 1975. In this case, the land is efficiently used during each time period when densities adjust costlessly and when no speculation exists.

The case of efficient over time use of land has been studied by Fujita [3], 1973, in a dynamic version of the Herbert-Stevens model in which the demand for housing is known over time for the various housing types. The case of short-run inefficient use of land has been studied by Anas [1], 1974; the urban markets are in equilibrium at the short-run under the assumption that the old urban households are <u>not</u> myopic over space although they are myopic over time. His model presents problems of consistency: it assumes a uniform utility level in the short-run, although such an assumption is a static long-run equilibrium assumption used in the nondynamic literature.

Further, in his generalization of one shock to multiple shocks of the same magnitude and frequency, the myopia over time assumption is an uncomforting one, in view of the previous discussion.

The point that this paper makes is that the methodology in trying to model dynamic behavior of economic agents in an urban context depends on the type of shock, its timing, the vintage of the urban residents and their state of learninc and, most crucially, it depends on the time after the shock that the snapshot is to be taken.

The Base Period; Long-Run Equilibrium (LRE)

Before the shock is encountered in the closed city by changing its level of the labor force, the urban economy is assumed to have reached LRE. According to the standard model, in that city all of the households (represented by one worker) are identical in preferences, with no initial holdings and with a utility function defined over the urban residential land, q, occupied by each household and all other urban commodities, z, consumed:

$$U = u(z,q)$$

where U is increasing in both arguments at a decreasing rate. The budget constraint is of the form:

$$z(x) + p(x)q(x) + T(x) = w$$

where p, T, and w are the price of land, transport costs and the wage rate, correspondingly. For reasons of notational simplicity, no time subscript would imply hereon value during the base time period; x is a subscript of distance from the center. The wage rate is formed in the aggregate production sector according to the standard conditions of marginal product valuation. There is congestion externality in the public good (transportation) sector in this city and the congestion level at each ring depends on the amount of transport capacity provided (here being just the land used in the transportation sector) and the spatial distribution of the population. The form of the transport cost function incurred at each ring by a worker crossing that ring for the journey to work is similar to Mills' [4], 1971:

$$T'(x) = \tau + \rho_1 \left[\frac{N_{(x)}}{L_T(x)}\right]^{\rho_2}$$

where, $T'(x)$ is the transport cost incurred at ring x, L_T is the amount of land used for transportation and τ, ρ_1, ρ_2, parameters. The model does not incorporate capital into the analysis (this can easily be done as an extension), and, indeed, one of its aims is to show that it is mostly the structure of the model that produces the interesting results, rather than the acknowledgment of capital and its properties in the analysis. This, of course, does not diminish the additional richness of insight if capital is incorporated.

Each commuter is paying under market equilibrium, the average social cost of transporting himself to the center, and, thus, the congestion externality is not priced. The total transportation costs incurred by a worker residing at x are:

$$T(x) = \int_x^{\bar{x}^o} \left\{\tau + \rho_1 \left[\frac{N_{(y)}}{L_T(y)}\right]^{\rho_2}\right\} dy$$

and, all transport receipts have to equal the transport expenditures by a Transportation Authority:

$$\int_{x^\infty}^{\bar{x}^o} T(x)\ [-N'(x)]dx = \int_{x^\infty}^{\bar{x}^o} p(x)L_T(x)dx$$

where, $\bar{x}^o$ is the (given) CBD boundary; x^∞ is the city boundary to be endogenously computed. If we assume no cross subsidy by residential ring, i.e., constant returns to scale in the provision of transport capacity, then:

(1) $T(x)\ [-N'(x)] = p(x)L_T(x)$

The first order condition for optimization (subscript stands for partial derivative with respect to the subscript and prime stands for partial derivative with respect to distance) is:

$$\frac{U_q}{U_z} = p$$

According to the standard analysis, the spatial equilibrium condition <u>during LRE</u> is:

$$(2)\quad p'(x) = -\frac{1}{q(x)} \cdot T'(x)$$

which results from setting the partial derivative of the Lagrangean with respect to x equal to zero.

All households are located, thus the following conditions have to hold:

$$N(\bar{x}^{\circ}) = \bar{N}$$

$$N(x^{\infty}) = 0$$

and all land at each ring is occupied:

$$(3)\quad \frac{2\pi x dx - L_T(x)}{q(x)} = -N'(x)$$

From (1), (2), and (3):

$$\frac{p'}{p} = -\frac{T'(x)}{T(x)} \cdot \frac{L_T(x)}{2\pi x dx - L_T(x)}$$

i.e., the percent change of the price of land is proportional to the percent change of the transport cost and the amount of land used for transportation over the amount of land used for residential ratio.

Every household at LRE enjoys a utility level U, independent of distance, so that:

$$\frac{U_z}{U_q} = \frac{q(x)}{w - [T(x) + z(x)]} = \frac{L_T(x)}{T(x)[-N'(x)]}$$

From the above:

$$\therefore \quad \frac{2\pi x dx - L_T(x)}{w - [T(x) + z(x)]} = \frac{L_T(x)}{T(x)}$$

and if by L_H we denote the total amount of land for housing at each x:

$$\therefore \quad \frac{L_H(x)}{L_T(x)} = \frac{p(x)q(x)}{T_x}$$

or, the amount of land for housing L_H over the amount of land in transportation equal to individual housing expenditures to the transport expenditures ratio.

From (2):

$$\frac{p'}{p} = \frac{T'(x)}{p(x)q(x)}$$

or:

the rate of change of the price of land is equal to the ratio of the transport costs incurred at location over the housing expenditures. Other market equilibrium conditions require that, $p(x^{\infty}) = R_{AGR}$, i.e., the price of land for residential (and transport) use at the periphery has to equal the agricultural opportunity cost of the land, at LRE. In the short-run the residential densities do not change:

$$-N'(x,t) = -N'(x)$$

$$L_H(x,t) = L_H(x)$$

$$L_T(x,t) = L_T(x)$$

and the transportation costs of each residential ring of the old city increase according to:

$$T'(x,t) - T'(x) = \rho_1 \left\{ \left[\frac{N(x,t)}{L_T(x,t)}\right]^{\rho_2} - \left[\frac{N(x)}{L_T(x)}\right]^{\rho_2} \right\} =$$

$$\rho_1/L_T(x) \left\{ (N(x) + \Delta N)^{\rho_2} - N(x)^{\rho_2} \right\}$$

The rent that the transportation authority now is willing to pay for the occupation of the land to the landowners is:

$$T(x,t)\ [-N'(x,t)] = p(x,t)\ L_T(x,t)$$

or

$$T(x,t)\ [-N'(x)] = p(x,t)\ L_T(x)$$

and in comparison to the base year:

$$T(x,t) > T(x) \quad \Rightarrow p(x,t) > p(x)$$

so that:

$$\frac{T(x,t)}{T(x)} = \frac{p(x,t)}{p(x)}$$

However, this bid-rent for land used in the transportation sector, to be designated as $p_T(x,t)$ herein, may or may not be identical to its opportunity cost, i.e., its price for residential use at time t.

The above $p_T(x,t)$ is the maximum price the landowners are able to get for their land that has been committed to transport use in the short-run(t). The relative magnitude of this price over distance to a LRE price after the shock will indicate if undersupply or oversupply of land for transportation occurs in the short-run, depending if the p_T is higher or lower, correspondingly.

Turning our attention to the residential side, in the short-run, the old urban households will now be acting as monopsonists; in the short-run t under the myopic assumption mode, they do not maximize their utility over time and/or space but at the location they presently occupy by just changing their consumption of all other urban commodities z and by exhausting their current budget. Thus, the first order conditions for utility maximizations do not hold any longer. If the landowners quote a rent in excess of the one paid in the base year and the households are not able to incur it, then it will be assumed that these households just leave their old residence and they exit the land market. This will occur if some new movers are willing to incur the new rent and move in; this in turn will depend on the alternative consumption pattern that the new residents will have in the new residential ring at the old urban area's fringe, as well as the resulting utility levels.

The new urban residents (their behavior described in the Appendix) are to enjoy a uniform utility level U^t if located in the new residential ring extended between x^∞ and $x^\infty(t)$. However, given a utility level U^t and a transport cost schedule $T(x,t)$ over all distances x they are able to bid for residential land at any other distance within the old residential ring $x^\circ \leq x \leq x^\infty$.

This bid-rent $\hat{p}(x,t)$ is derived from:

$$\frac{\bar{U}^t_{\hat{q}}}{\bar{U}^t_{\hat{z}}} = \hat{p}$$

where $\hat{z}$ and $\hat{q}$ are such that:

$$\hat{z}(x,t) + \hat{p}(x,t)\,\hat{q}(x,t) + T(x,t) = w(t)$$

and:

$$U^t(x,t) = \bar{U}^t$$

and $\hat{z} \geq 0$ in this ring $x^\infty \leq x \leq x^\infty(t)$.

Thus, the new residents are competing according to the above scheme for residential land with the old residents, having the option of just residing in the urban fringe. In the case of $U^t \equiv U$ (i.e., identical individuals moving in the city), and due to the lower wage rate assumption at t from the base year, it follows that the utility level $\bar{U}^t$ is less than $\bar{U}$. However, the total land expenditures $\hat{p}(x,t)\ \hat{q}(x,t)$ may or may not exceed the old level $p(x)\ q(x)$. As densities remain constant in the short-run in the residential rings in the old section of the city that has been partitioned into lots of a fixed size $q(x)$ and these sites do not change at t, it is the yield of the lot size in general that is of interest to the landowners of the old section of the city; the new residents are able to pay an amount equal to $\hat{p}(x,t)\ \hat{q}(x,t)$ for locating at an old x versus an amount $q(x)\ p(x)$ that old residents there were willing to pay if:

$$\hat{p}(x,t)\ \hat{q}(x,t) > p(x)\ q(x)$$

then the following possibilities exist:

1: $\hat{q} > q$; $\hat{p} < p$

2: $\hat{q} > q$; $\hat{p} > p$

3: $\hat{q} < q$; $\hat{p} > p$

Any of the above may occur at any part of the old ring, with possibly more than one case occurring in the old part of the city. No. 1 case is clear: the old residents outbid the new ones in the old locations and the "shock" does not have any effect, at these locations, on the price of land.

In case of No. 2, the existing densities would drive the newcomer with different preference functions to lower his bid-rent, decreasing the amount of land demanded to the lot size supplied only if the form of the preference function is such that:

$$\frac{U^t_q}{U^t_{\tilde{z}}} = \tilde{p}(x,t) \qquad \tilde{z}(x,t) + \tilde{p}(x,t)\ q(x) + T(x,t) = w(t)$$

and

$$\tilde{U}(x,t) > \bar{U}^t \ ; \ \tilde{p} > p$$

when:

$$\tilde{U}^t = u^t[\tilde{z}(x,t),\ q(x)]$$

If $\tilde{U}(x,t) < \bar{U}^t$ then the $\tilde{p}$ is an artificial bid that no newcomer is willing to incur, and it has no effect on the market rent function, even if $\tilde{p} > p$. Of course, the above does not hold for the newcomers if they have the same preference function as the old residents.

In case No. 3, newcomers with preference functions different from the old residents, the existing densities would lead the newcomer to lower his bid-rent and increase the amount of land demanded to the lot size applied, if:

$$\frac{U_q^t}{U_{\tilde{\tilde{z}}}^t} = \tilde{\tilde{p}}(x,t)$$

$$\tilde{\tilde{z}}(x,t) + \tilde{\tilde{p}}(x,t)\ q(x) + T(x,t) = w(t)$$

$$\tilde{\tilde{U}}(x,t) > \bar{U}^t$$

$$\tilde{\tilde{p}} > p$$

where: $\tilde{\tilde{U}}^t = u^t[\tilde{\tilde{z}}(x,t),\ q(x)]$

It should be noted that cases 2 and 3 cannot both occur in the same city at the same time. Of course, if $\hat{p}(x,t)\ \hat{q}(x,t) < p(x)\ q(x)$ then the shock does not have any impact at this particular x.

If $\hat{p}(x,t)\ \hat{q}(x,t) < p(x)\ q(x)$ then the price quoted in the short-run is the same as the price being paid in the base year; this is sometimes referred to as the stickiness phenomenon in the land prices.

Assuming that new prices are quoted and incurred by the new households, the utility level enjoyed by the old residents is derived from:

$$U(t,x) = u[\hat{\hat{z}}(x,t),\ q(x)]$$

where:

$$\hat{\hat{z}}(x,t) + \hat{\hat{p}}(x,t)\ q(x) + T(x,t) = w(t)$$

and:

$$U'(t,x) \neq 0$$

To observe the differential decrease over distance of the utility level of the old residents, since the q remains the same in the short-run as in the base year at each location, the change in the utility level has to be a function of the relative change in the consumption of the composite commodity over the base year; i.e.,

if: $\frac{\partial}{\partial x}\left[z(x) - \hat{\hat{z}}(x,t)\right] \lesseqgtr 0$

then: $U'(t,x) \gtreqless 0$

assuming that U is a monotonically increasing function of z.

Of course the interesting question is, at what location (closer to or further out from the center) does the utility level decline more rapidly (since a decrease of the wage rate will always imply a decrease of all old urban residents' utility). Assuming that all new residents locate in the new residential ring $x^{\infty} \leq x \leq x^{\infty}(t)$ and the old residents outbid the new residents at the old section of the city (i.e., the old residents pay the same land price as during LRE) the budget constraint will be:

$$z^*(x,t) + p(x)\ q(x) + T(x,t) = w(t)$$

where z* is the new level of composite commodity consumed. Examining the expression:

$$\frac{\partial}{\partial x}\left\{z(x) - z^*(x,t)\right\} \lesseqgtr 0$$

to determine whether the utility increases or decreases with distance:

$$\frac{\partial}{\partial x}\left\{w - T(x) - w(t) + T(x,t)\right\} =$$

$$T'(x,t) - T'(x) = \rho_1\left\{\left[\frac{N(x,t)}{L_T(x)}\right]^{\rho_2} - \left[\frac{N(x)}{L_T(x)}\right]^{\rho_2}\right\} > 0$$

Thus, the utility level in the short-run for this case decreases with distance, always being lower than during LRE. This is an important finding since it says that the difference in utility depends on the differential in congestion tolls (which is directly affected by the newcomers), and it further suggests that individuals would be motivated to move inwards when congestion (toll) increases. If the newcomers outbid the old residents in some portion of the old section of the city, under case No. 2, p. 6, this has to be the neighboring section of the new ring (Figure 1). If by x we denote the distance from the center beyond which the old residents have to adjust their rent payments, then the two areas ($\bar{x}^{\circ} \leq x \leq \underset{\sim}{x}$; $\underset{\sim}{x} \leq x \leq x^{\infty}$) are experiencing two different patterns of utility decrease. In $\bar{x}^{\circ} \leq x \leq \underset{\sim}{x}$ the lowest utility level is higher than or equal to the highest utility level of any household in the ring: $\underset{\sim}{x} \leq x \leq x^{\infty}$.

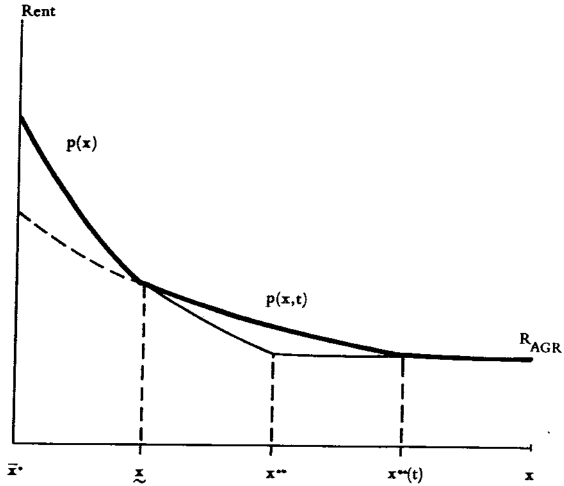

Figure 1. The new residents have to outbid the old residents at the ring $\underset{\sim}{x}$ x^{**}, if at any ring in the old section of the city.

The utility is also decreasing with distance in the second ring ($\underset{\sim}{x} \leq x \leq x^{\infty}$), because:

$$\frac{\partial}{\partial x} \left\{ z(x) - z^*(x,t) \right\} =$$

$$\frac{\partial}{\partial x} \left\{ w - T(x) - p(x)\; q(x) - w(t) + T(x,t) + \tilde{p}(x,t)\; q(x) \right\} =$$

$$T'(x,t) - T'(x) + \frac{\partial}{\partial x} \left[q(x) \left\{ \tilde{p}(x,t) - p(x) \right\} \right]$$

which has a ubiquitous sign, given that the difference in prices has to increase beyond $\underset{\sim}{x}$, and the q is an increasing function of x. It is also deduced from the above that the drop in utility is more drastic at any $\overline{\overline{x}}$ in $\underset{\sim}{x} \leq x \leq x^{\infty}$ than in any $\overline{\overline{\overline{x}}}$ in $\bar{x}^{\circ} \leq x \leq \underset{\sim}{x}$ if:

$$T'(\overline{\overline{x}},t) - T'(\overline{\overline{x}}) + T'(\overline{\overline{\overline{x}}}) - T'(\overline{\overline{\overline{x}}},t) > \frac{\partial}{\partial x} \left[q(\overline{\overline{\overline{x}}}) \left\{ \tilde{p}(\overline{\overline{\overline{x}}},t) - p(\overline{\overline{\overline{x}}}) \right\} \right]$$

and for the particular case of $\rho_2 = 2$:

$$\rho_1 \Delta N \left[\Delta N \left\{ \frac{1}{L_T^2(\overline{\overline{x}})} - \frac{1}{L_T^2(\overline{\overline{\overline{x}}})} \right\} + 2 \left\{ \frac{N(\overline{\overline{x}})}{L_T^2(\overline{\overline{x}})} - \frac{N(\overline{\overline{\overline{x}}})}{L_T^2(\overline{\overline{\overline{x}}})} \right\} \right] >$$

$$\frac{\partial}{\partial x} \left[q(\overline{\overline{\overline{x}}}) \left\{ \tilde{p}(\overline{\overline{\overline{x}}},t) - p(\overline{\overline{\overline{x}}}) \right\} \right]$$

Conclusions

The paper put forth the conditions surrounding the short-run effects of a sudden (first) disturbance of an urban economy and examined its form the "day after." The analysis is rather general since it does not employ a specific utility function, it incorporates congestion effects and, finally, allows for more than one type of urban household (the new migrants may or may not be characterized by the same preference function). The purposes of the paper were to describe the myopic behavior of households and landowners and to provide an alternative conceptual framework towards deriving a theory of urban disturbances and urban agents' "learning" behavior as intrinsic elements of a dynamic theory of urban spatial structure, with agents of different vintages.

An interesting conclusion of the paper is that, in the short-run, the utility level of old residents decreases with distance as new migrants move into the urban setting, due to relative increases in the congestion toll, that would imply that as the congestion level increases, households would be motivated to move to more central locations.

Although the analysis was performed in absence of capital (assuming, however, that land indivisibility is mostly due to the various capital improvements on it), by incorporating it into the analysis one would extend the present study. However, the most interesting extension would be a detailed examination of a second (random?) shock and the establishment of conditions of expectancy by the various agents, by establishing a functional relationship between the ΔN and its effects on each residential ring.

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APPENDIX

The new residents, to reside further than the x^{∞} up to $x^{\infty}(t)$, are to be distributed so as to maximize their utility (provide by a preference function either the same as or different from the old residents) subject to their wage constraint (uniform for old and new residents), resulting to:

for $x^{\infty} \leq x \leq x^{\infty}(t)$

A.1 $$\frac{U_z^t}{U_q^t} = \frac{1}{p(x,t)}$$

A.2 $$z(x,t) + p(x,t)\ q(x,t) + T(x,t) = w(t)$$

A.3 $$T(x,t) = \int_x^{x^{\infty}} \left\{\tau + \rho_1 \left(\frac{N(y,t)}{L_T(y,t)}\right)^{\rho_2}\right\} dy$$

and, all transport receipts to cover the transport expenditures in the new rings:

A.4 $$\int_{x^{\infty}(t)}^{x^{\infty}} T(x,t)[-N'(x,t)]dx = \int_{x^{\infty}(t)}^{x^{\infty}} p(x,t)L_T(x,t)dx$$

and, again, assuming no cross subsidization:

A.5 $$T(x)[-N'(x,t)] = p(x,t)L_T(x,t)$$

under the equilibrium condition:

A.6 $$p'(x,t) = -\frac{1}{q(x,t)} \cdot T'(x,t)$$

All new residents are to be located in the new ring:

A.7 $$\begin{cases} N(x^{\infty}(t),t) = 0 \\ N(x^{\infty}) = \Delta N \end{cases}$$

and all new residents, if to be located in the new ring, are to enjoy a uniform utility level:

A.8 $$U^t(x,t) = \bar{U}^t$$

for

$x^{\infty} \leq x \leq x^{\infty}(t)$

And, again:

$$\text{A.9} \quad p(x^{\infty}(t),t) = R_{AGR}$$

assuming that the agricultural rent is invariant with time (no speculation in the land market). In case of identical preference functions between the new and the old residents, the

$$\text{A.10} \quad \bar{U}(t) < \bar{U}$$

if the uniform wage rate (t) is decreasing with the labor force influx (i.e., there are decreasing returns to scale with respect to labor in the production side).