PRACTICAL APPLICATIONS OF AN INTERREGIONAL MULTIPLIER MODEL*

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When the economy of a region is subjected to changes in the level of activity of one or more of its sectors, both the real structure of the regional economy and the monetary flow of receipts and expenditures between it and the other regions affect the character of the response to that change. The eventual effects on income, employment, and output resulting from any multiplier stimulus are determined, in large measure, by the structural composition of the region's industrial sector. Regions incapable of supplying the necessary inputs into production resulting from changes in final demand suffer the inevitability of experiencing multiplier leakages, spillover and reduced feedback effects. The economy of Scotland, as a case in point, is a highly specialized economy depending heavily on a few select industries for export production, and on numerous import flows to satisfy its industry and final demands.[1] These trading flows have the effect of inextricably binding the production processes of Scotland to the rest of the United Kingdom; with variations in either having repercussions on the other.

The present paper attempts to analyse the quantitative significance of production-trading dependencies on a regional economy, using Miller's [3] [4] adaption of the conventional matrix multiplier as the analytic operative. The paper is divided into four parts. Part I seeks to derive a regional technical coefficient matrix from the national counterpart, using a limited number of available or readily estimatable regional control totals. Part II incorporates the statistical input derived in Part I in focussing upon the construction of an interregional system of production-trading flows. Equipped with this information, Part III empirically tests the Miller interregional multiplier model. In conclusion, Part IV concentrates upon an evaluation of the results of this testing, and seeks to draw a few conclusions concerning the efficacy of this technique as an operational tool of region analysis.

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[1]Although the models to be developed have been generalized, the empirical testing of the models was carried out using the regional economy of Scotland as an integral component of the national economy of the United Kingdom.

I. Regional Production Coefficient Estimation

Perhaps the most limiting constraint on effective regional analysis arises out of the glaring paucity of regional statistical information pertaining to production functions and trading flows. Because of the formidable barriers experienced by regional input-output practitioners, efforts aimed at circumventing or surmounting these seemingly intractable problems have given rise to a profusion of techniques tailored to estimate regional reaction paths.[2] This section follows in their wake. That is, an effort will be made in this section to mathematically simulate a regional matrix of technical coefficients.

The regional technical coefficient matrix simulation procedure to be used is based upon the Richard Stone technique for updating input-output coefficients [1]. In its simplest form the regional adaption of this technique requires three sets of data: (i) a national input-output matrix; (ii) a conformable vector of regional gross outputs; and (iii) control totals of regional intermediate inputs and outputs for the desired year. To illustrate this technique, denote the national technical coefficient matrix by A_n, the vector of regional gross outputs by q_r, and the vectors of intermediate output and input by c_r and d_r respectively. The intermediate control vectors can be obtained from estimates of regional gross outputs, final demand, and primary inputs by the following relationships

(1) $c_r \equiv q_r - f_r$

and

(2) $d_r \equiv q_r - z_r$

where f_r denotes the vector of regional final outputs and z_r denotes the vector of regional primary inputs.

After having derived c_r and d_r from q_r, f_r, and z_r, the next step is to estimate the coefficient matrix of the regional economy A_r. In essence, this entails revising the national coefficient matrix in line with regional control totals through a process of scaling the rows and columns of a derived regional transactions table to agree with the intermediate totals of outputs and inputs. These scaling adjustments in absorption flows, hence technical coefficients, are based on the assumption that national-regional differences in technical coefficients are due to main factors (i) price differences, (ii) substitution effects resulting in changes in factor inputs, and (iii) differences in production techniques that influence all of the elements in a given column. On the further assumption that the second factor operates uniformly along its columns, the problem and its solution can be formulated as follows.

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[2] For example, see Hoch [2], Moore and Petersen [5], and Moses [6].

In adjusting the national coefficients to conform with the desired regional specifications, the first step is to take into account differences in national and regional price levels. This is easily accomplished by adjusting A_n to the price levels of the regional economy through the following calculation

(3) $\bar{A} = \hat{p} A_n \hat{p}^{-1}$

where $\hat{p}$ is a diagonal matrix of price relatives (regional output prices divided by the corresponding national price levels).

The second source of disparity in the regional and national coefficient matrices is assumed to be that of differential patterns of factor input use. For example, regional production functions are often characterized by definite locational preferences for fuel, whether it be natural gas, petroleum products, coal, or electricity. If differences of this nature proceed in a uniform fashion across all of the regional industries under examination, then this behavior can be corrected for by multiplying the rows of the price adjusted regional transactions table by the elements of a diagonal matrix $\hat{r}$ which will be greater than one for regional inputs with a positive bias and less than unity if the regional industry is less utilized as a factor input than is true of the national level as a whole.

Concomitant with this factor substitution effect, differences in the respective production processes will probably arise out of national-regional diversities in the industrial contributions of value added per unit of output. This condition can easily arise if either level, regional or national, employs a more labor or capital intensive form of production than the other. Or, perhaps one region may absorb more highly processed inputs than is the case in the other region with the result being differential levels of value added per unit of output. This tendency can be corrected for by multiplying the columns of $\bar{A}$ by the elements of a diagonal matrix $\hat{s}$ which would be greater than unity if the regional per unit output levels of value added were less than the national level, and less than unity if the opposite condition prevailed. The formal incorporation of these influences into the model yields the following result

(4) $\bar{\bar{A}} = \hat{r} \hat{p} A_n \hat{p}^{-1} \hat{s}$

Thus, from a knowledge of the differences in price relatives and estimates of the regional total intermediate sales and purchases, the elements of the r and s scaling diagonals can be estimated. Algebraically, the national technical matrix transformation proceeds as follows. Again, letting A_n denote the known matrix of technical coefficients, $\hat{p}$ the price diagonal whose elements are the ratios of prices in the region to prices nationally, the problem becomes one of solving for the unknown diagonal matrices $\hat{r}$ and $\hat{s}$. On the assumptions made,

(5) $\bar{\bar{A}} = \hat{r} \hat{p} A_n \hat{p}^{-1} \hat{s}$

(6) $= \hat{r} A^* \hat{s}$

where $A^* \equiv \hat{p}\, A_n\, \hat{p}^{-1}$. Assuming that one has access to information of the regional intermediate output vector, u say, an intermediate input vector, v say, and a vector of total outputs q_r, then

(7) $$A_r\, q_r = u$$

and

(8) $$\hat{q}\, A_r'\, i = v$$

One can make an initial estimate, u_o say, of u by remultiplying q_r by A^*. Thus

(9) $$A^*\, q = u_o$$

In general, however, $u_o \neq u$, but one can force an equality by an appropriate multiplication of the rows of A^*. Thus,

(10) $$(\hat{u}\, \hat{u}_o^{-1}\, A^*)\, q_r = u$$

If one regards the term in brackets as an estimate of A_r(the regional coefficient matrix), it can be seen that it satisfies the row conditions but not the column conditions. These can be satisfied by a substitution for the row adjusted A matrix from (10) into (8) followed by an appropriate multiplication of the columns of A^*. Thus,

(11) $$\hat{q}\, A^{*\prime}\, \hat{u}\, \hat{u}_o^{-1}\, i = v_o$$

and

(12) $$\hat{q}\, (\hat{v}\, \hat{v}_o^{-1}\, A^{*\prime}\, \hat{u}\, \hat{u}_o^{-1})\, i = v$$

Again, regarding the bracketed term as readjusted estimate of A_r, one can see that it now satisfies the column conditions but not, in general, the row conditions. However, by an iterative repetition of this cycle of operations one will eventually obtain a convergence that satisfies the row and column conditions. Thus, after n + 1 iterations, the following is obtained,

(13) $$(\hat{u}^{n+1}\, \hat{u}_o^{-1}\, \ldots\ldots\, \hat{u}_n^{-1}\, A^*\, \hat{v}^{n+1}\, \hat{v}_o^{-1}\, \ldots\ldots\, \hat{v}_n^{-1})\, q_r = u_{n+1}$$

For a sufficiently large n, the term in brackets can be taken as an estimate of the regional coefficient matrix A_r.

II. Estimation of an Interregional Model

The derived regional coefficient matrix calculated above does not differ from that prepared for the country as a whole in any fundamental respect. However, the interregional system, the basis of which is the regional input-output system, is quite different in substance. Whereas a national table shows only the interrelationships between industries within a country, a regional model can be constructed to show two things: first, the interrelationships between all industries within a single region, and second, the interrelationships between industries in that region and those in all other regions. In a regional model exports to other regions of the same country (as opposed to exports overseas), instead of being shown in aggregate as one constituent of total final demand, can be broken down according to the industry in the importing regions which makes ultimate use of them. Similarly, the region's imports from other regions can be broken down according to the industry in which they are used.

A regional input-output model therefore attempts to define the relationships between industries in different parts of the country in statistical terms. In its simplest form it shows the relationship between just two regions (e.g. Scotland and the rest of the United Kingdom), but in theory there is no reason why it should not be extended to permit the analysis of several regions at the same time; the only real problem being one of data availability. In a two-region model the overall input-output table will have four constituent parts. The first will show the flow of goods from industries in region A to other industries in region A; the second will illustrate the flow of goods from industries in region A to industries in region B; the third the flow of goods from industries in region B to industries in region A, and the fourth the flow of goods from industries in region B to other industries in region B.

The first problem in deriving such a table is the statistical one of garnering sufficient information to estimate the inter-industry relationships both within regions and between regions. If actual information was a necessary condition of regional input-output formulation, then this type of study would rarely achieve fruition. In an attempt to circumvent this barrier to completion, the following study made one rather restrictive assumption. It was assumed that purchases by the regional industries (Scottish) were made from the same regional sources (Scottish) to the fullest extent possible. In other words, it was assumed that there was no interregional movement of final demand goods except in those cases where the output of a given industry in one region was less than the output of that industry implied by the final demands of that region.[3]

The next step was to allocate Scotland's intermediate outputs and inputs by using industry, supplying industry, and region. This was done as follows. The intermediate output of each Scottish industry was allocated amongst the same region's industry groups, and similarly, Scotland's intermediate inputs into Scottish industries were allocated amongst the supplying industries. This yielded a regional matrix whose elements contained two value flows (an output flow and an input flow). In a typical case the output flow (x) showed

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[3]This assumption which has been common to a number of similar studies, realizes the same implications as the locational preference assumption in the Moore and Peterson [5] study.

the flow of goods from industry i in Scotland in industry j in Scotland and the rest of the United Kingdom. The input flow (y) showed the flow of goods into industry j in Scotland from i in Scotland and the rest of the United Kingdom. The difference between y and x could therefore be taken to represent the interregional import into industry j in Scotland from industry i in the rest of the United Kingdom.

Each of the Scotland-to-rest of the United Kingdom quadrant of the interregional input-output table was therefore arrived at by identifying every case where the x value exceeded the y value, with the resulting difference (x - y) value taken to be a net export to the rest of the United Kingdom. Similarly, each element of the rest of the United Kingdom-to-Scotland quadrant of the table was arrived at by identifying every case where the y exceeded the x value and the difference being defined as the net import into a Scottish industry from the identical rest of the United Kingdom industry. The Scotland-to-Scotland elements of the interregional table were, of course, equal in every case to the smaller of the two values, x and y. The intraregional flows of the rest of the United Kingdom were estimated by subtracting from each element in the transaction flow table for the United Kingdom as a whole the sum of the corresponding element in the Scotland-to-Scotland quadrant, and the corresponding element in either the Scotland-to-rest of the United Kingdom quadrant or the rest of the United Kingdom-to-Scotland quadrant. This method of constructing the table meant that there was no possibility of there being a sale both from Scottish industry i to the rest of the United Kingdom industry j and from the rest of the United Kingdom industry j to the Scottish industry i.

The mathematical derivation of the interregional table proceeded as follows. The object was to formulate a table in the following general form

$$(14) \qquad R = \left[\begin{array}{c|c} R_{11} & R_{12} \\ \hline R_{21} & R_{22} \end{array}\right]$$

where: the suffixes 1 and 2 refer to Scotland and the Rest of the United Kingdom respectively; the order of R_{pq} is determined by the number of industries being considered, with the flows from region p to region q (p,q = 1,2).

Let ${}_{pq}r_{ij}$ represent a typical element of the matrix R_{pq}. The problem then is to estimate these elements using the information given in Part I. The initial task is to apportion the Scottish total intermediate inputs and outputs amongst the various industry groups.

First, consider the total intermediate outputs in Scotland. By forming a vector a_r such that

$$(15) \qquad a_r = A_r \, i$$

where i is a unit vector, the elements of the resulting vector reflect the summed total of the output coefficients for such industry. By further forming a vector b_r such that the elements of that vector represent the reciprocal of the summed output coefficients of each industry,

(16) $b_{ri} = 1 / a_{ri}$ $(i=1, \ldots, n)$

the resulting vector provides a basis for determining the proportionate weightings of the respective output coefficients. When the diagonal of the b_r vector is postmultiplied by the regional output technical coefficient matrix A_r, the resulting matrix yields a matrix of coefficients whose size reflects the relative importance of each receiving industry's absorption input. That is,

(17) $B_r = \hat{b}_r A_r$

Then, by forming a matrix A_r one is able to show how the total intermediate output produced in Scotland is used in the various industries in the United Kingdom as a whole,

(18) $A_r = \hat{u} B_r$

where the $\hat{u}$ represents the diagonal matrix of Scottish intermediate outputs. This last formulation represents a matrix of output flows based upon the schema of output coefficient determination. A typical element of this matrix is $a_{r.ij}$.

Second, consider the total intermediate inputs in Scotland. The procedure used is very similar to that just described except that the input columns have been transposed. Again, form a vector such that the elements represent the summed value of input coefficients,

(19) $\acute{a}_c = \acute{i} A_r$

Form another vector such that the elements denote the summed values of the respective industry's input coefficients, that is, for a vector b_c such that

(20) $b_{ci} = 1 / a_{ci}$ $(i=1, \ldots, n)$

The premultiplication of the diagonal of this latter vector by the regional coefficient matrix yields a weighted matrix of input coefficients

(21) $B_c = A_r \hat{b}_c$

Finally, form a matrix A_c which shows how the total intermediate inputs used in Scotland have been purchased from the various industry groups in the United Kingdom as a whole.

(22) $A_c = B_c \hat{v}$

where $\hat{v}$ represents the diagonal of the vector of intermediate input totals for Scotland. A typical element of this matrix can be denoted by $a_{c.ij}$.

The allocation of the elements of matrix R_{pq} proceeds as follows. First, consider the elements ${}_{11}r_{ij}$ of submatrix R_{11}. By definition, the elements of this submatrix are determined by the minimum elements of either the input or output flows. That is,

$$(23) \qquad {}_{11}r_{ij} = \min (a_{r.ij}, a_{c.ij})$$

If the element $a_{r.ij} > a_{c.ij}$ then Scotland is considered a net exporter of industry i's output to the rest of the United Kingdom. Thus, ${}_{12}r_{ij} = a_{r.ij} - a_{c.ij}$, and ${}_{21}r_{ij} = 0$. In other words, if Scotland's intermediate output exceeds Scotland's intermediate input needs, then the balance is exported to the rest of the United Kingdom. Also by definition, trading flows are in net balances and therefore a region cannot be both an importer and exporter of the same industry's output.

On the other hand, if $a_{r.ij} < a_{c.ij}$ then Scotland is a net importer of commodity i from the rest of the United Kingdom. Thus, ${}_{21}r_{ij} = a_{r.ij} - a_{c.ij}$, with ${}_{12}r_{ij} = 0$. The last possibility exists when $a_{r.ij} = a_{r.ij}$, which means that on balance Scotland neither exports nor imports that industry's output. As a final note, in all cases ${}_{12}r_{ij}$ and ${}_{21}r_{ij}$ shows net exports from and imports to Scotland respectively. Thus, at least one of ${}_{21}r_{ij}$ and ${}_{12}r_{ij}$ must equal zero, for all i, j.

The intra-rest of the United Kingdom submatrix is obtained from the national transactions flow matrix (X_{ij}) in the following way:

$$(24) \qquad \text{let } m_{ij} = {}_{11}r_{ij} + {}_{21}r_{ij} + {}_{12}r_{ij} = \max (a_{r.ij}, a_{c.ij})$$

then,

$$(25) \qquad {}_{22}r_{ij} = X_{ij} - m_{ij} \quad \text{all } i, j$$

This concludes the construction of the two sector interregional production-training model.

III. An Intranational Multiplier Model

The conventional techniques used in estimating regional multipliers usually take the form of merely scaling national multiplier coefficients by predetermined regional scalars, variants of industrial composition analysis, economic base studies, or the formulating of interregional trade multipliers along Keynesian multiplier lines. Each constitutes a look at one or more facets of the functioning and structure of a systems complex. This complex of systems may be viewed either horizontally as an intricate network of regions, each in itself a system, or vertically as a nonadditive overlay of interregional systems of money flows, commodity flows, population flows, industrial locations, etc., reflecting the spatial configuration of resources, technological development, etc., and such motives as efficiency and welfare. However conceived, each approach yields a unique and often rewarding insight into the regional multiplier process.

The idea of structural interdependence amongst regions has been commented upon by a number of economists, most notably Moses, Isard, Leontief, Moore, Peterson, and Miller. However, most of the comment has been strictly theoretical or conjectural in nature. Thus, while the need for more rigorous treatment of actual trading flows has been duly acknowledged, it has been conspicuously neglected in the area of applied research. The explanation is not far to seek. In all probability no other area of applied economic research experiences the degree of stringency in its statistical constraints as does the area of interregional trade analysis. However, if one is willing to consider the implementation of a model such as that developed in Parts I and II, then it becomes possible to explore this interesting dimension of regional analysis.

The technique to be used to evaluate the multiplier implications of an interregional model is based upon Miller's model which breaks down the conventional set of simultaneous input-output equations into two matrix equations:

$$(26) \qquad (I - R_{11})\, X_1 - R_{12}\, X_2 = Y_1$$

$$-R_{21}\, X_1 + (I - R_{22}) X_2 = Y_2$$

where X_i and Y_i refer to the respective levels of gross output and final demand. Pursuing the logic of this specification further, the total output of industries within Scotland can be divided into production requirements needed to satisfy the final demands of the rest of the United Kingdom as well as those requirements necessary to facilitate the satisfaction of purely Scottish final demands. Strictly speaking, final demands in this context must be somewhat redefined. The export part of final demand has been reduced, in each case, to shipments to the other region on final demand account alone. However, the fact that the model is intended to consider the multiplier implications of changes in final demand and not final demand in aggregate permits reference to "final demand" in this rather loose manner.

Thus, regular simultaneous equation procedures on (26) give the following results:

$$X_1 = \left[(I - R_{11}) - R_{12}\, (I - R_{22})^{-1} R_{21}\right]^{-1} \left(Y_1 + R_{12}\, (I - R_{11})^{-1}\, Y_2\right)$$

$$X_2 = \left[(I - R_{22}) - R_{21}\, (I - R_{11})^{-1} R_{12}\right]^{-1} \left(Y_2 + R_{21}\, (I - R_{11})^{-1}\, Y_1\right)$$

This model permits a more accurate delineation of the respective multiplier processes than is possible under the more conventional scalar multiplier formulations. That is, the final demand changes can be directed to either region. Unlike the conventional matrix multiplier techniques whose regional application usually rests upon an assumed constraint of an imposed uniform distribution of multiplier effects according to certain specified proportional characteristics, this model injects a condition of realism by allowing each region to experience the natural polarization tendencies inherent in their production patterns.

IV. Empirical Testing

The results contained in Tables 1 and 2 have been derived from an empirical testing of the Miller model, with an autonomous change (unit increase) in final demand being directed separately at both the Scottish and rest of the United Kingdom regions. Because of the constancy of input coefficients and the linearity of output functions, it clearly wouldn't have mattered whether the system was solved twice, once with the new total requirements vector and once with the old total requirements vector or whether one worked from the beginning with a final demand vector which recorded only changes in each vector.

The advantages gained through the usage of this multiplier model are both substantial and numerous. Unlike the traditional scalar multipliers with their generalized applications and assumed distributive implications, this model permits the incorporation of each regional industry's unique technology into a model capable of analysing the respective regional impacts. The ability to direct the production stimulus to the confines of a particular region, rather than rely upon some criterion of multiplier impact distribution, is perhaps its greatest advantage. If definite agglomeration economies do exist, then the resulting polarization of the multiplier impact must, to a certain extent, vitiate many of the plausible regional inferences to be drawn from an adaption of a national matrix multiplier.

To take a general example, if the expected rates of growth of final demand over a period of years for the country as a whole are known, then one should be able, with input-output analysis, to translate these rates of growth into expected rates of growth for each industry in each region. This is something which the other techniques previously alluded to patently cannot do at the present time. It is not enough to take a national target and allocate it to regions on the basis of employment or population or some other arbitrary indicator. This has been known to produce disastrously inaccurate results. Nor is it enough, of course, to merely wait for the changes to occur before providing for them; there is no surer recipe for waste and inefficiency.

In addition, regional input-output enables the indirect effects of possible government policies aimed at developing areas to be examined. If the government were considering the introduction of a new factory into a declining area, for example, there is at the moment no way in which one can systematically assess the impact of this new factory on other industries in the area, on the level of employment or on the economy of the region generally except through a technique such as that developed by Miller. These indirect effects can often be just as important as the immediate effects; indeed, the overall effects of certain types of expansion outside a region may well be greater than some types of expansion within that region. An input-output table, properly structured to consider these interregional flows, can provide the analytical framework necessary for an examination of these indirect effects. At the same time, due acknowledgement must be made to the inherent limitations of the technique. Apart from the fact that it implies the availability of accurate data -- which are difficult to obtain -- a crucial assumption built into the whole system is that the technological relationships implied by the coefficients in the table are those actually operating over the period in question. Unforeseen

TABLE 1: Rest of the United Kingdom Gross Output Multipliers, Temporal Reaction Sequences

	Industry	Simple Gross Output Multiplier (1)	First Order Multiplier Reaction (2)	Cumulative Second Order Multiplier Reaction (3)	Cumulative Third Order Multiplier Reaction (4)	Cumulative Fourth Order Multiplier Reaction (5)	Multiplier Limit (6)
1.	Coal Mining	1.359	1.170	1.258	1.305	1.330	1.339
		.125	.003	.003	.003	.003	.003
2.	Stone and Slate	1.597	1.212	1.356	1.439	1.501	1.554
	Quarrying and Mining	.018	.002	.002	.003	.003	.003
3.	Chalk, Clay, Sand and	1.266	1.126	1.182	1.216	1.240	1.261
	Gravel Extraction	.019	.005	.006	.006	.006	.006
4.	Miscellaneous Mining	1.324	1.199	1.261	1.291	1.309	1.319
	Industries	.060	.006	.007	.008	.008	.008
5.	Grain Milling	2.703	1.660	2.080	2.345	2.512	2.642
		.192	.029	.034	.036	.038	.039
6.	Bread and Flour	2.107	1.431	1.714	1.895	2.012	2.107
	Confectionery	.196	.033	.039	.043	.045	.046
7.	Biscuits	2.247	1.427	1.690	1.851	1.954	2.037
		.337	.160	.186	.203	.213	.217
8.	Bacon Curing, Meat	2.427	1.580	1.918	2.114	2.228	2.292
	and Fish Products	.310	.108	.130	.141	.148	.151
9.	Milk Products	3.224	1.747	2.291	2.676	2.944	3.204
		.233	.012	.015	.017	.019	.020
10.	Sugar	3.568	1.797	2.407	2.869	3.217	3.309
		.437	.206	.271	.319	.356	.377
11.	Cocoa, Chocolate and	2.279	1.500	1.809	2.009	2.144	2.226
	Sugar Confectionery	.157	.033	.042	.049	.054	.057
12.	Fruit and Vegetable	2.398	1.555	1.885	2.081	2.200	2.278
	Products	.156	.023	.029	.033	.036	.037

TABLE 2: Scotland's Industry Gross Output Multipliers, Temporal Reaction Sequences

Industry	Simple Gross Output Multiplier (1)	First Order Multiplier Reaction (2)	Cumulative Second Order Multiplier Reaction (3)	Cumulative Third Order Multiplier Reaction (4)	Cumulative Fourth Order Multiplier Reaction (5)	Multiplier Limit (6)
1. Coal Mining	1.359	.044	.052	.059	.063	.067
	.125	1.170	1.250	1.279	1.292	1.307
2. Stone and Slate	1.597	.365	.439	.501	.548	.562
Quarrying and Mining	.018	1.002	1.002	1.002	1.003	1.004
3. Chalk, Clay, Sand and	1.266	.182	.216	.240	.258	.267
Gravel Extraction	.019	1.003	1.005	1.006	1.006	1.006
4. Miscellaneous Mining	1.324	.170	.191	.204	.212	.217
Industries	.060	1.080	1.102	1.111	1.116	1.119
5. Grain Milling	2.703	.618	.769	.864	.925	.946
	.192	1.444	1.639	1.726	1.766	1.788
6. Bread and Flour	2.107	.257	.322	.364	.392	.426
Confectionery	.196	1.440	1.629	1.719	1.766	1.794
7. Biscuits	2.247	.233	.286	.320	.341	.356
	.337	1.497	1.706	1.816	1.880	1.929
8. Bacon Curing, Meat	2.427	.110	.135	.152	.163	.168
and Fish Products	.310	1.610	1.954	2.144	2.248	2.298
9. Milk Products	3.224	.572	.735	.847	.924	.989
	.233	1.619	1.975	2.179	2.295	2.388
10. Sugar	3.568	.039	.047	.053	.058	.062
	.437	1.822	2.471	2.976	3.368	3.602
11. Cocoa, Chocolate and	2.279	.577	.718	.813	.878	.846
Sugar Confectionery	.157	1.207	1.299	1.356	1.395	1.428
12. Fruit and Vegetable	2.398	.498	.607	.673	.714	.741
Products	.156	1.343	1.478	1.546	1.586	1.603

technical change, and even changes in output levels, may make nonsense of this. One can only adopt an empirical attitude of these things -- to satisfy oneself about the validity of the technique by experimentation which discovers how well it works in practice.

A Miller-type multiplier model, within its limitations, is determined by three differential regional reaction paths: that associated with the diverse industrial compositions of regions; that associated with the integration of heterogeneous as well as homogeneous market and supply areas of an economy; and where regional data exists, that associated with different production practices and consumption patterns in the respective regions. Via these three reaction paths, the impact of any regional (and thus national) change is transmitted to other regions, affecting their structure in terms of output, employment and income.

Column one of Tables 1 and 2 illustrates the gross output multipliers resulting from a unit change in the final demands of the respective regional industries. The multiplier determinant in each case was the Leontief inverse $(I - A^{1963})^{-1}$, which calculated the total multiplier effect for each industry in turn.[4] The Scottish multiplier was derived by scaling each change in the national multiplier by a diagonal matrix of Scottish production performance coefficients before the changes in national gross domestic output were summed. The upper coefficient alongside each industry specification denotes the national industry multiplier, with the lower value reflecting the "derived" Scottish multiplier effect. Implicit in the derivation of the Scottish multiplier is the assumption that the multiplier impacts are distributed regionally according to a predetermined set of production performance coefficients.[5]

The main weakness of the above regional multiplier method is that no consideration is taken of the fact that the locus of stimulation is an important determinant of the purely regional impact. Whenever a change in local final demand does take place, the forces of agglomeration economies and transportation costs will inevitably manifest themselves in a polarization of the resulting multiplier effects. The Miller multiplier model as developed is capable of isolating the respective intraregional and extraregional effects. In addition it offers an opportunity to select the locus of initial stimulation through being capable of facilitating independent changes in either region's final demand. A comparative static aspect of this multiplier is developed in the remaining portions of Tables 1 and 2, where the successive stage effect of the multiplier process has been estimated using an interatively determined power series approximation of the matrix multiplier operative.

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[4]The empirical analysis of Scottish and Rest of the United Kingdom industries focussed upon calculating multiplier coefficients for an economic structure disaggregated into one hundred and twenty industries. Tables 1 and 2 present a sampling of the results, with the complete set available from the author.

[5]The production performance coefficients were derived by rationing Scotland's industry output levels over the corresponding national total.

Column two of Tables 1 and 2 exhibits the rest of the United Kingdom and Scottish direct multipliers resulting from independent unit changes in the respective regional final demand vectors. As with column one the upper coefficient value in each table represents the rest of the United Kingdom impact, with the lower multiplier denoting the Scottish reaction impact. Depending on the region of initial stimulation, the multipliers are primarily the result of intraregional production flows, or of leakage and spillover effects. Although of negligible significance in the Scottish multiplier case, the feed back following an initial multiplier leakage is of considerable importance in the rest of the United Kingdom case.

Focussing attention on either Tables 1 or 2 it is evident that there are differential rates of convergence to the eventual multiplier limit. Each multiplier begins with column two which represents the direct inputs into production, with the third column representing the cumulative, first plus second, order reactions of the multiplier process, and so on. Implicit in this technique is the assumption that individual regional industry production reactions to real changes in level of their final demand are uniform across the two regional sectors being analysed. This means that industries are assumed to replenish their depleted inventories at approximately the same speed, with the direct plus indirect demands resulting from each round of multiplier reactions being transmitted across both sectors in uniform fashion. If one is willing to accept the tenability of this assumption, then the development of each multiplier takes on an added dimension of significance.

The economic implications of this type of analytic approach to the regional multiplier process are obvious; not only is it possible to determine the magnitude of the true regional multipliers, but it is also feasible to gauge the approximate length of the respective reaction impact periods, the iterative speed of the leakage effect, and the size of the spillover resulting from the directed stimulation of the respective regional industries. This approach is particularly useful in analysing the multipliers of relatively small and specialized regional economies, such as Scotland's, where the leakage effects are significant in many instances. Although Tables 1 and 2 each represent the convergence trends of the respective regional multipliers, there is one very important difference. In Table 1 the initial multiplier stimulation, i.e., change in final demand, takes place within the rest of the United Kingdom sector, whereas in Table 2 it is Scotland which experiences the incremental change in final demand in the initial instance.

Examining both Tables 1 and 2 for comparative purposes, a number of interesting observations emerge. First, the traditional scaling technique of gauging the regional effects of a nationally determined multiplier, column one, is seen to either exaggerate or underestimate the regional multipliers depending on the locus of stimulation. As was previously mentioned, this technique assumes that there is a uniform distribution by area of each multiplier effect, with regional production capacities determining regional multiplier implications. However, in failing to incorporate the inevitable polarization tendencies of a directed multiplier, the resulting multiplier coefficients always suffer from a directional bias. This is particularly true for the specialized regional economy surrounded by an effective transportation buffer, since existing regional industries tend to experience definite locational preferences whereas the unbalanced structure of the manufacturing sector results in immediate leakage effects. In the first case the regional multipliers tend to be high since the multiplier reactions are restricted largely to the confines of the regional sector, whereas in the latter case the multiplier is low since the leakage is immediate and often significant in size.

In illustrating the importance of being able to supply the necessary inputs into production, an examination of the distilling industry multipliers within the rest of the United Kingdom table, and the mineral oil refining coefficients within the Scottish table is particularly revealing. If the final demand of the distilling industry within the rest of the United Kingdom is increased by one unit, the leakages inherent with the regional production process result in the Scottish multiplier being twice that of the rest of the United Kingdom multiplier, despite the regional choice of initial stimulation. The rest of the United Kingdom distilling multiplier is 1.537, of which 1.000 results from the postulated change in final demand, whereas the Scottish distilling industry multiplier is 1.082, all of which is the result of experienced spillover and multiplier effects. In contrast, a unit increase in the Scottish final demand for mineral oil results in the total multiplier effect, aside from the initial change, being siphoned off to the rest of the United Kingdom sector. This is the direct result of Scotland not having a mineral oil refining industry of significant size within its industrial structure.

A further perusal of Tables 1 and 2 highlights a number of conclusions which can be drawn concerning the size of the respective multipliers. First, the direct rest of the United Kingdom multiplier is usually significantly larger than the comparable Scottish multiplier. The average rest of the United Kingdom multiplier is 1.872 whereas the comparable Scottish multiplier is only 1.451. The differential becomes even more exaggerated when the respective leakage effects are considered. The average leakage resulting from a directed rest of the United Kingdom multiplier is .052 with the inclusion of the distilling and jute industries, and only .034 when these anomalies are excluded in calculating the leakage effect. On the other hand, a final demand change directed at the Scottish sector results in an average leakage of .520. The leakage effects are more meaningfully interpreted if the unit increase in the total multiplier effect is not taken into consideration in calculating the multiplier impacts. In this case, the rest of the United Kingdom multiplier resulting from an initial increase in the Scottish vector of final demands is, on the average, greater than the multiplier effect experienced by the Scottish sector itself. In contrast, the rest of the United Kingdom leakage is negligible relative to the total multiplier limit.[6]

In summary, aside from the information that can be gained through an examination of the individual multipliers, the broader outlines of the complete model present an effective method of regional development analysis. First, the model can be used with sufficient operational skill to permit fairly accurate forecasts of induced changes in economic activity. In being able to determine the size, speed, and spillover effects of individual industry multipliers, the technique shows promise of being able to improve the manipulation of short-term regional-cyclical policy. Secondly, through being able to calculate regional leakage effects, the regional policy-maker is permitted an opportunity to analyse the extent to which higher governmental expenditures give rise to (a) higher local production and further disbursements of income locally, or (b) higher imports from other regions. Thirdly, the method affords ground for favouring regional development along one line rather than another. Through successive replacements of the import coefficients within each regional structural matrix, it is possible to select those industries with the greatest effect on the regional economy as a whole. Depending on the degree of import coefficient displacement desired, the policy-maker is permitted an opportunity to investigate the possible effects on other local industries of establishing or expanding a particular industry in a region.

In spite of the many advantages of this type of multiplier analysis, any final determination of its efficacy must still be qualified by the assumptions which underlie its development. If, however, one is willing to accept the tenability of reasonably stable production-trading coefficients through time, and a uniform lag in production reactions, then this analytic method must emerge as one of the more promising methods of regional analysis. Perhaps its greatest attribute is that it is capable of analysing specialized regional economies which, to date, have had to suffer the implications of crude, and often inaccurate, methods of regional analysis.

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[6]If consideration is to be given to international trade leakage effects, an often important component of British production processes, then the multiplier induced changes in the individual industry gross domestic output levels should, in turn, be scaled by a diagonal vector of import coefficients. These coefficients could be derived by rationing the level of imports into manufacturing over the total gross output levels of the respective industries. In the Scottish case, for example, the strictly intraregional flows would remain the same, however, the inclusion of the import leakage would have an effect similar to a scaled diminution in the respective trade-production submatrices.

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