URBAN LAND USE MODELS: SOLUTION METHODS

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A land use model as used in urban planning, particularly urban transportation planning, is a set of mathematical procedures used to allocate or distribute region-wide projections to spatial sub-areas, and predict land use consumption implications of the various activities distributed. Region-wide projections of population and economic activity are made for some future point in time, and land use models are used to actually allocate the increments of growth in population, employment, etc. to the various sub-areas of the region. Land use models use region-wide projections as given for input to a process by which predicted change or growth will be distributed to the sub-areas. Similarly, the region-wide projections might have come from a national allocation. This paper is concerned with allocating metropolitan region projections to sub-areas of the region.

The purpose of this paper is to review alternative deterministic solution methods for the allocation process--allocation between economic sectors, allocation through time, and allocation to the spatial sub-areas. Emphasis is given to differences in modeling strategies between two methods of allocation between economic sectors--simultaneous solution versus sequential solution.

This discussion focuses on the class of models that Harris [3] terms *deterministic*, as contrasted to those he terms *probabilistic*. By the nature of operational urban land use models this discussion also focuses on *macro* models and not *micro* models. Therefore, this paper does not purport to provide a broad review of the urban land use modeling literature. It discusses alternative strategies incorporated in deterministic, macro models for allocating future estimates of activities to sub-areas.

A short preparatory discussion of theory, aggregation, and data in land use modeling sets the stage for discussing allocation between activity sectors, allocation through time, and allocation to spatial sub-areas.

Adaptation of Micro-Economic Theory to Land Use Models

The basic problem in land use modeling is to adapt micro-economic choice

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theory in the land and housing market to land use models. Relating the supply of sites to the demand for sites is the process to be modeled. Most models of urban location behavior are static in the sense that they are concerned with location choice at a particular point in time. They are concerned with equilibrium, optimizing choice in the short run with a large number of variable taken as given. However, for forecasting purposes and for allocative purposes it may be important to know how spatial patterns change, how neighborhoods change, how densities change, and where people and businesses move. Land use models must be made dynamic to account for changes over time, or static descriptive models of existing spatial patterns are assumed to remain valid for the future.

Aggregation

Another major problem in adapting micro-economic market theory to land use models involves data requirements. Choice of variables, construction of mathematical models, calibration of the model to the metropolitan area involved----all of these steps are conditioned by the extent to which small-area data are available or the extent, cost, and speed of generating it. The degree of aggregation, of course, plays a crucial role. Many kinds of data are simply unavailable for areal units smaller than census tracts. Lack of disaggregation tends to make a model "rough-grained" in the sense that it may fit the data well but tell us little. On the other hand, too much disaggregation may be possible in the sense that processing and effectively using all available data may be extremely costly.

Allocating establishments (including households) to sites is the actual process by which urban spatial patterns result. The real estate "market place" where establishments are allocated to sites or parcels of land is an extremely complex process to model, and would, at the disaggregate establishment-site level be overwhelming. As Lowry [8] points out, land use modelers generally collapse like individual establishments to aggregate activity sectors, and collapse individual sites to districts or sub-areas of the region. By means of aggregation the problem is transformed from one of allocating individual establishments to individual sites to one of allocating activities to sub-areas. Thus, the level of aggregation is an important and explicit part of land use modeling.

Data

As important as the degree of aggregation is the nature of the variables used. Some variables, such as employment by place of work are difficult to obtain by small areas. In addition some variables such as the physical and social environment of a neighborhood, types of amenities, level of public services available are difficult to quantify. In such cases, not only is the level of aggregation a problem, but the choice of a surrogate variable or the construction of a numberical index must also be made.

However, the final data problem may be the most difficult one of all.

Any forecasting model must be so constructed that the independent variables used to estimate dependent variables must themselves be variables that can be predicted for the future. That is, the inputs to the forecasting model must also be forecast. Perhaps land value is to be used as an independent locating variable. Assume that, among other things, the number of households of a certain type locating in a particular sub-area is a function of the land values in that sub-area. One of the major sources of possible error becomes the accuracy with which land values for that particular sub-area can be forecasted to various points in the future. In areal units that contain primarily undeveloped land which may become used for residential purposes in the future, it is inordinately difficult to forecast variables such as the land values, level of public services which will eventually be provided, or various social and physical variables which relate to the characteristics of an areal unit at some future point in time. Exceptions are those variables that planners have control over, i.e. policy related variables, as contrasted to those that are uncontrollable by policy and are a function of private decisions.

Sectoral Allocation

In terms of the overall purposes of a land use model residential location or industrial location are but one part of the overall process. The breakdown into component parts of the overall model tends to be treated differently depending on the particular study. Allocation between activity sectors within a time period can be simultaneous or sequential. Simultaneous solutions are predicted on the assumption that the various kinds of activities--industrial, commercial, residential, etc.--are interdependently located within the sub-areas, whereas sequential solutions assumes an ordering of locational decisions take place. Both models are assumed to operate within time frames of sufficient length to allow the sequential or interdependent processes to take place. Five to ten year intervals are normally used, which allows enough time for inter-sectoral influences such as employment upon residential location.

A simultaneous solution is illustrated in the following simplified example. Simultaneous solutions with regard to land use models means the various economic and residential sectors within a sub-area are highly interrelated and are jointly determined.

(1) $Y_1 = f(Y_2,X)$[1] (basic employment equation)

$Y_2 = f(Y_1,Y_3,X)$ (residential equation)

$Y_3 = f(Y_1,Y_2,X)$ (local service equation)

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[1]Read "Y_1 as a function of Y_2 and X"

Let Y_1 refer to the level of basic employment, Y_2 to the level of local service employment, Y_3 the residential population and X refers to independent variables or lagged variables. With three equations and three unknowns (Y_1,Y_2, and Y_3) the equation set can be solved simultaneously for each time period.

The Empiric Model [4] that was initially used in the Boston Regional Planning Project is formulated as a set of simultaneous equations. Although a certain number of independent variables are included, the major feature of this model is that nine dependent variables, four categories of households stratified by income and five categories of employment, are simultaneously solved for at once.

In addition to the Empiric Model, Lakshmanan uses a simultaneous solution in his models for Calgary [1] and Connecticut [6]. Within a shift-share framework of economic analysis, Lakshmanan simultaneously solves for activity sectoral change.

A sequential solution is used in many land use models and assumes an ordering of location decision. Basic employment is exogenously located, households locate in proximity to places of work, and local service employment locates in relationship to population like:

(2) $Y_1 = f(X)$ (basic employment equation)

$Y_2 = f(Y_1,X)$ (residential equation)

$Y_3 = f(Y_2,X)$ (local service employment equation)

The equations are solved in sequence which effects a location for local service employment that is dependent upon the location of residential population, which in turn, is dependent upon the location of basic employment.

There exist a variety of models that can be classified as sequentially relating economic sectors. The Lowry Model [7] and its derivatives--TOMM [2] and BASS [5]--best illustrate this sequential approach.

The sequential modeling strategy assumes that the local service employment also generates households to be located, and that the sequential model must iterate through the cycle of residential population generating demand for local services whose employees must be located, who in turn generate demand for local services, and so on. Diagrammatically, this is illustrated in Figure 1. Each iteration of the steps enclosed in the loop of Figure 1 locates a smaller number of households until the employment needed to serve those households becomes negligible.

This sequential approach does not necessarily reopen the basic and non-basic economic base argument, but assumes some employment levels and locations to be exogenously determined and are not a function of serving local population. On the other hand, local service employment does service local population and its level and location is determined by the location of population.

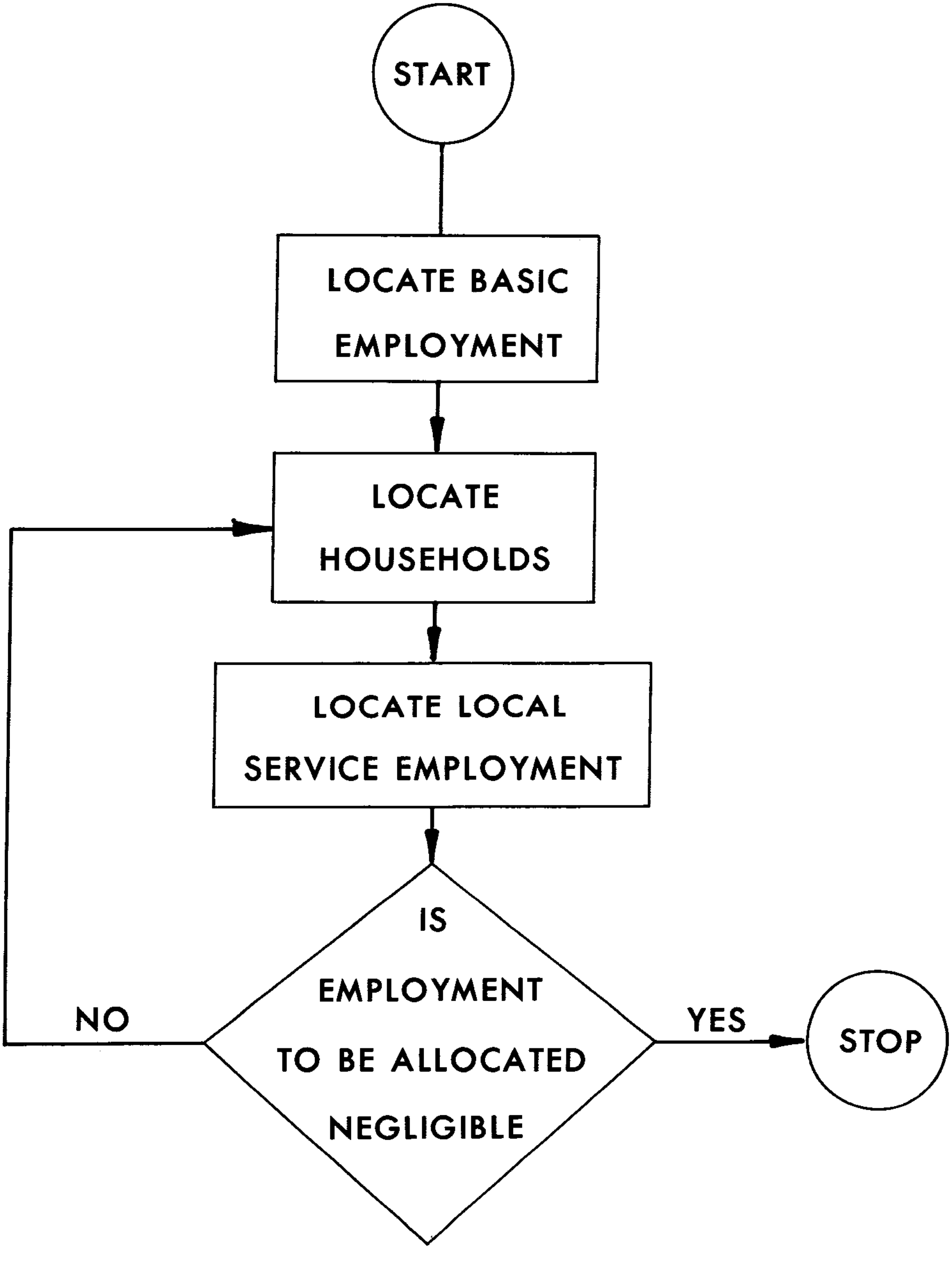

FIGURE 1. Iterations of location employment and households

In BASS, output from the employment sub-model serves as input to the residential sub-model. Incremental additions of employment are considered to be functions of already calculated employment change.

There is one exception to the above statements about the BASS model. Three classifications of service employment--educational services, construction, and agriculture are all allocated after residential development. The overall effect on residential and employment distributions, though, seems to be rather negligible. Such a provision in the model, of course, does not alter the fact that the solution is sequential. Normally the succession of models is represented as having some relationship to the sequence in which actual development takes place and to the lags which are observed in the real world. In BASS the implicit assumption is that the location of work places (other than those for agriculture, education, and construction) are not determined by residential growth but rather residential growth is determined by newly located employment centers.

Treatment of Time

Regardless of whether the solution methods for sectoral interrelationships are simultaneously or sequentially solved, a land use model must also allocate population and economic activity to small areas for some future time. The process by which the urban system moves from the initial to its terminal state must be specified.

However, as implied in the above discussion of sectoral allocation, the time period within which activities are sequentially or simultaneously allocated must be long enough for the sector interrelationships and influences to take place. Many model builders use recursive progressions where the output from one cycle becomes the input to the next cycle:

$$
\begin{aligned}
(3) \qquad Y_t &= f(Y_0) \\
Y_{t+1} &= f(Y_t) \\
Y_{t+2} &= f(Y_{t+1}) \\
&\vdots \\
Y_{t+n} &= f(Y_{t+n-1})
\end{aligned}
$$

In this system of difference equations, the behavior of the function at given time period depends on past behavior.

Alternatively a static model can be used of the form:

$$
(4) \qquad Y_t = f(X_t)
$$

and a future estimate of X will provide an estimate of Y, i.e.:

$$
Y_{t+n} = f(X_{t+n})
$$

In this latter case the behavior of the function has nothing to do with the past behavior of the function.

Spatial Allocation

The actual allocation of activities to sub-areas can be handled in a variety of ways. Spatial allocation is necessary because the sub-areas used in land use models are too small to assume their independence. That is, employment growth in one sub-area will have an impact on more than that one sub-area. Workers will likely live in other sub-areas and in turn will shop in yet another. Thus, the problem is to allocate activities among sub-areas.

One popular approach is to use accessibility measure to determine a relative attractiveness measure for each sub-area. Accessibility of sub-area j to employment opportunities A^E_j can be expressed as:

(5) $$A^E_j = \sum_{k=1}^{n} E_k \, / \, t^b_{jk}$$

i.e. the employment (E_k) for a sub-area k divided by the travel time between sub-area j and that (k) sub-area, summed over all sub-areas. The parameter b generally varies from approximately one to a value of three depending on the type of travel--work, shopping, etc. The accessibility measure works in a way that gives sub-areas close to other activities, in this case employment, a higher value because the travel times are smaller, conversely, a sub-area that is farther from employment opportunities will be divided many times by larger travel times, which will result in a lower accessibility.

When using accessibilities for allocation, activities are then allocated to a sub-area in proportion to the sub-areas accessibility vis-a-vis other sub-area. Operationally, relative accessibility RA^E_j can be expressed as:

(6) $$RA^E_j = A^E_j \, / \sum_{j=1}^{n} A^E_j$$

which expresses the proportional share for sub-area j, based solely on accessibility.

Another approach is to compute attractiveness scores for sub-areas which take into consideration more than accessibility. This is accomplished by weighing and combining sub-area characteristics such as amount and quality of vacant land, nearness to schools, availability of urban services, and existing employment. For example in locating motor freight transportation and warehousing, and wholesale trade industries in the San Francisco Bay Area, BASS computes a score for each sub-area located on a freeway or state highway:

$$S_j = -0.36X_j^6 \; -0.19X_j^{10} +0.15X_j^{15} +0.18X_j^{17} +0.74X_j^{21} -0.34X_j^{24} -0.35X_j^{33} +0.30X_j^{34} +0.70X_j^{39} \quad (7)$$

Where:

X_j^6 = sub-area j employment in group #6 industries (furniture, paper, petroleum, plastics, rubber, and misc. manufacturing)

X_j^{10} = sub-area j employment in group #10 industries (electrical machinery, and professional, scientific instruments)

X_j^{15} = sub-area j employment in group #15 industries (finance, insurance and real estate)

X_j^{17} = sub-area j employment in group #17 industries (repair and misc. services)

X_j^{21} = sub-area employment in military

X_j^{24} = density of development in sub-area j

X_j^{33} = group 14 employment (eating and drinking establishments and hotels) within 30 minutes of sub-area j

X_j^{34} = vacant plus agricultural land in sub-area j

X_j^{39} = group 6 employment within 30 minutes of sub-area j

An average sized firm is located within the sub-area having the largest score. The score for that sub-area is then reduced, and another firm is located in the sub-area which now has the highest score. By reducing the score of a sub-area after locating a firm there, the sub-area may lose its highest position and the subsequent locating firm will go elsewhere.

Future Activity Levels in Sub-Areas

The above discussion treats each element of the allocation process separately. First the concern was allocation between activity sectors. Then time was treated, and finally spatial allocation was discussed. This section attempts to combine these elements of the land use modeling process.

Let Y_{ijt} be the level of activity i in sub-area j at time t, the present. The problem is to estimate the level of activity i in sub-area j at time t + n, the target year.

To illustrate the combined allocation the following three equation simplified model is suggested. The first equation is:

$$(8) \qquad Y_{1jt} = f(Y_{1j0}, A^{P}_{j0}, X_{jt})$$

which can be interpreted as the basic employment in sub-area j at time t (Y_{1jt}) is a function of the initial basic employment in sub-area j at time 0 (Y_{1j0}), a function of the accessibility of sub-area j to population at time 0 (A^{P}_{j0}) and also a function of some independent variables in sub-area at time t (X_{jt}).

The second equation is:

$$(9) \qquad Y_{2jt} = f(Y_{1jt}, Y_{3jt}, Y_{2j0}, A^{E}_{j0}, X_{jt})$$

which can be interpreted as the population in sub-area j at time t (Y_{2jt}) is a function of the basic employment in sub-area j at time t (Y_{1jt}), a function of the local service employment in sub-area j at time t (Y_{3jt}), a function of the initial population in sub-area j at time 0 (Y_{2j0}), a function of the accessibility of sub-area j to employment at time 0 (A^{E}_{j0}), and a function of some independent variables in sub-area j at time t (X_{jt}).

The third equation is:

$$(10) \qquad Y_{3jt} = f(Y_{1jt}, Y_{2jt}, Y_{3j0}, A^{P}_{j0}, X_{jt})$$

which is interpreted as the local service employment in sub-area j at time t (Y_{3jt}), a function of the basic employment in sub-area j at time t (Y_{1jt}), a function of population (Y_{2jt}), a function of the initial local service employment (Y_{3j0}), a function of the accessibility of sub-area j to population (A^{P}_{j0}), a function of some independent variables in sub-area j at time t (X_{jt}).

This pedagogical model is set up for simultaneous solution and for recursive progressions where the output from one cycle becomes the input to the next time period. It is formulated to use accessibility as the means to perform the spatial allocation.

The basic employment equation becomes:

$$(11) \qquad Y_{1jt+1} = f(Y_{1jt}, A^{P}_{jt}, X_{jt+1})$$

when estimating the level of economic activity for the first time increment. At the final stage the local service equation becomes:

$$(12) \qquad Y_{3jt+n} = f(Y_{1jt+n}, Y_{2jt+n}, Y_{3jt+n-1}, A^{P}_{jt+n-1}, X_{jt+n})$$

Closing Note

The allocation of region-wide projections of population and economic activities to sub-area is a difficult but necessary process in land use and transportation planning. Sub-area forecasts are central inputs to land use planning and transportation planning models. There is much confusion surrounding the mechanics of most land use models, yet there is a great deal of similarity in the kinds of allocation that must be performed and in the way in which allocation is performed using land use models. This discussion isolated the various allocative elements--sectoral, temporal, and spatial--and illustrated how each is usually handled in deterministic urban land use models. Although each land use model is unique they all contain methods to allocate activities between sectors, over time, and over space. The allocative elements were then combined in a simple illustrative model.

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